Two chambers, one valve

$$\dot{m} = \sqrt{\Delta p}$$
$$\dot{\Delta p} = -k\dot{m}$$

A) Calculating the oscillation:

$$\Delta\Delta p = -k\Delta t\sqrt{\Delta p}$$

Which will be stable ($\Delta p \to 0$) if:

$$|\Delta p'| < |\Delta p|$$
$$|\Delta p + \Delta\Delta p| < |\Delta p|$$
$$\left|\Delta p - k\Delta t\sqrt{\Delta p}\right| < |\Delta p|$$

Let $\Delta p > 0$:

$$k\Delta t\sqrt{\Delta p} < 2\Delta p$$

So the system well be stable if:

$$\Delta t < \frac{2\sqrt{\Delta p}}{k}$$

Which only holds for all Δp, if $\Delta t \to 0$, which of course, due to simulation with discrete time steps, isn't possible.

The amplitude of oscillation:

$$\Delta p' = -\Delta p$$

$$k\Delta t\sqrt{\Delta p} = 2\Delta p$$

$$\Delta p = \left(\frac{k\Delta t}{2}\right)^2 \equiv \Delta p^*$$

B) Solution:
If $\Delta p > \Delta p^*$, we don't have a problem, otherwise let

$$\dot{m} = f(\Delta p)\sqrt{\Delta p}$$

Using the above calculations, the requirement for stability:

$$k\Delta t f(\Delta p)\sqrt{\Delta p} < 2\Delta p$$

So we must have such a function, for which:

$$f(\Delta p) < \frac{2\sqrt{\Delta p}}{k\Delta t}\left(= \left|\frac{1}{\frac{d\Delta\Delta p_{base\ case}}{d\Delta p}}\right| = \frac{1}{\frac{d\left(k\Delta t\sqrt{\Delta p}\right)}{d\Delta p}}\right)$$

For example, let

$$\boldsymbol{f}(\Delta\boldsymbol{p}) = \mathbf{2}\sqrt{\Delta\boldsymbol{p}} \left(= \frac{1}{\frac{d(\dot{m}_{base\ case})}{d\Delta p}} = \frac{1}{\frac{d(\sqrt{\Delta p})}{d\Delta p}} \right)$$

(The expressions in parenthesis are to give ideas for later use).

Which will stabilize the solution if:

$$2\sqrt{\Delta p} < \frac{2\sqrt{\Delta p}}{k\Delta t}$$

$$\boldsymbol{\Delta t} < \frac{\mathbf{1}}{\boldsymbol{k}}$$

We could've chosen any other good candidate for such a function, I don't know which function would be „the best". I suppose the closer $f(\Delta p)$ is to $\frac{2\sqrt{\Delta p}}{k\Delta t}$ the better, since such a function changes the original system the least. (For example, $f(\Delta p) = 0$ would work).
We could also chose a function so that the stability won't depend on the time step, for example

$$\boldsymbol{f}(\Delta\boldsymbol{p}) = \frac{\sqrt{\Delta\boldsymbol{p}}}{\boldsymbol{k\Delta t}}$$

(Which results from $\Delta\Delta p = -\Delta p$, that is we don't let Δp change it's sign).

Three chambers, two valves:

We need to start being more precise here:

$$\begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} \sqrt{|\Delta p_1|} * sgn(\Delta p_1) \\ \sqrt{|\Delta p_2|} * sgn(\Delta p_2) \end{bmatrix}$$

Let the chambers increase their pressure linearly with the air flow (as before), according to the constants k_1, k_2, k_3. And let the positive flow and pressure between the chambers be thought of as $CH1 \rightarrow CH2 \rightarrow CH3$. If so, we have:

$$\begin{bmatrix} \Delta\dot{p}_1 \\ \Delta\dot{p}_2 \end{bmatrix} = \begin{bmatrix} -(k_1 + k_2) & k_2 \cdot sgn(\dot{m}_1\dot{m}_2) \\ k_2 \cdot sgn(\dot{m}_1\dot{m}_2) & -(k_2 + k_3) \end{bmatrix} \begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix}$$

A) Calculating the oscillation:

$$\begin{bmatrix} \Delta\Delta p_1 \\ \Delta\Delta p_2 \end{bmatrix} = \Delta t \begin{bmatrix} \Delta\dot{p}_1 \\ \Delta\dot{p}_2 \end{bmatrix}$$

??? Condition for stability:

$$\left\| \begin{matrix} \Delta p_1 + \Delta\Delta p_1 \\ \Delta p_2 + \Delta\Delta p_2 \end{matrix} \right\| < \left\| \begin{matrix} \Delta p_1 \\ \Delta p_2 \end{matrix} \right\|$$

This seems to resemble the 1D case, and based on me playing around in Simulink also seems to be true. However, it may be too strong a condition, and a weaker one may also suffice.

To be continued…