Measuring Acceleration of Gravity To One Percent With a Video Camera
MCA Physics

Introduction

Galileo's Law of Falling Bodies states that in the absence of air resistance, all bodies near the surface of the earth accelerate downward at the same rate, known as the acceleration of gravity g, which is approximately 9.81 m/s/s. Later work has measured g to great precision and shown that it varies slightly (less than 1%) over the surface of the earth. Combining Galileo's law of falling bodies with elementary kinematics gives a simple equation for the vertical position of a falling body as a function of time: $y(t) = y_0 + v_{oy}*t - 0.5*g*t^2$, (Equation 1)
where y is the vertical position of the object, t is the time (in seconds), y_0 is the initial position of the object, v_{oy} is the initial vertical component of velocity, g is the acceleration of gravity, and t is the time.

One experimental challenge of a full experimental test of Eqn 1 is accurately fixing and determining the initial position, velocity, and time. It is hard to experimentally start an accurate clock at the precise instant an object is dropped. This experiment takes advantage of a property of parabolas (noting that the quadratic in equation 1 yields a parabola) that tells us that horizontal shifts (the t variable here), vertical shifts (the y variable here), and changes in the linear coefficient (v_{oy} here) do not effect the curvature of the parabola or (equivalently) the coefficient of the quadratic term, which here is $-0.5\ g$. So, even if we do not have the start time or position exactly right, if we obtain good data for height vs. time, the quadratic term in a best fit quadratic can still yield an accurate estimate for the acceleration of gravity g.
Hypothesis: With due experimental care, g can be determined with an accuracy of 1% or better watching a ball drop with a video camera and analyzing that data. The independent variable is the time, the dependent variable is the height. This data can be analyzed to determine an experimental value of g.

Method

A video camera (JVC Enviro) will record five trials of a 3/8" diameter steel sphere falling. To help stop the motion, the shutter speed is set to 1/4000 s, and artificial lighting provides enough light. The shiny sphere is set against a flat back background to provide contrast, and the camera is set to its highest resolution setting and oriented in "portrait" mode so the full 1920 pixel resolution is available to quantify the vertical motion. The distance between the shiny aluminum trim is carefully measured to be 845 mm which is used calibrate and provide a length scale in the analysis. The time scale is set to be the frame interval of 1/29.97 s.

After acquiring videos, each video is imported into the Tracker program for analysis. (https://physlets.org/tracker/) Videos are rotated into proper orientation, and a set of axes and a calibration stick are inserted. The calibration stick is then oriented vertically and stretched carefully from the top edge of the flat black area to the bottom edge of the flat black area, and the length of 845 mm is entered to provide the program with an accurate length calibration. Time between frames is 1/29.97 seconds, each point in time increments by that amount. The zeroth frame is reckoned to be the last frame before the dropped sphere moves in front of the black background. Then the position and time in each subsequent frame are recorded as the sphere moves through the black background.

After position vs. time are recorded for each trial, the data is put into a spreadsheet to convert the height into m so there are adjacent columns of height in m vs time. Then the two columns for each video are imported into Graph.exe to perform a least squares fit to a quadratic. The value of g for each video is then calculated to be -2*C, where C is the coefficient of the quadratic term in the fit.

Results (50 points)

Time (s)	Height (m)	Height (mm)
1/29.97		
2/29.97		
3/29.97		
4/29.97		
5/29.97		
6/29.97		
7/29.97		
8/29.97		
9/29.97		
10/29.97		

Table 1: Height vs. time for first video. Make additional copies in your notebook for additional trials.

Trial	C	g=-2*C (m/s)	REL ERROR
1			
2			
3			
4			
5			
AVERAGE			
STDEV			
SEM			
REL ERR			

Table 2: Values of quadratic coefficient and g for each video trial. REL ERROR is computed for each trial by using the accepted value of g = 9.805 m/s/s for the location. It is computed as (Experimental value – Accepted Value)/(Accepted Value). The bottom four rows are computed with standard spreadsheet function calls.

Your results section should contain a table and graph for each video analyzed in the lab, as well as a complete summary table (Table 2 above). Graphs should contain best fit quadratics with equations and R^2 displayed.

Discussion (50 Points)
Was the hypothesis supported? What were the largest and smallest relative errors for the five trails (in percent)? What is the relative error and SEM of the average g from all five trials? What are the essential elements of this experiment that allow for such an accurate determination of g?