PROBLEM SHEET 2 MATH2015 – DUE MONDAY 30 OCTOBER

1. If A is an $(n \times n)$-matrix and λ is a scalar, prove

$$\det(\lambda A) = \lambda^n \det(A)$$

directly from the definition of the determinant;

$$\det(A) = \begin{cases} \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(A(ij)) & n > 2 \\ a_{11}a_{22} - a_{12}a_{21} & n = 2 \end{cases}$$

[HINT: use induction i.e. show for $n=2$ first, then show that the statement is true if one assumes it is true for $((n-1)\times(n-1))$-matrices.]

2. Reduce the following matrix to Hermite form using elementary row operations assuming $ad-bc \neq 0$. Show each step clearly and simplify your answer.

$$\begin{pmatrix} a & b & f & 1 & 0 \\ c & d & g & 0 & 1 \end{pmatrix}$$

What do you notice about the last two columns? Use this observation to compute the inverse of

$$\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 3 \\ 2 & 4 & 2 \end{pmatrix}$$

Check your answer.