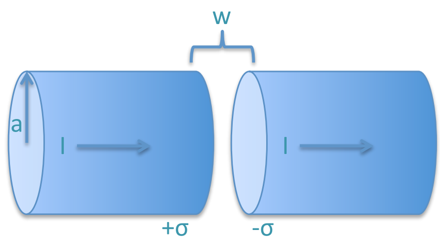
Assessed worksheet 4

Answers to be handed in at the physics departmental office (PevII 3A1) before:

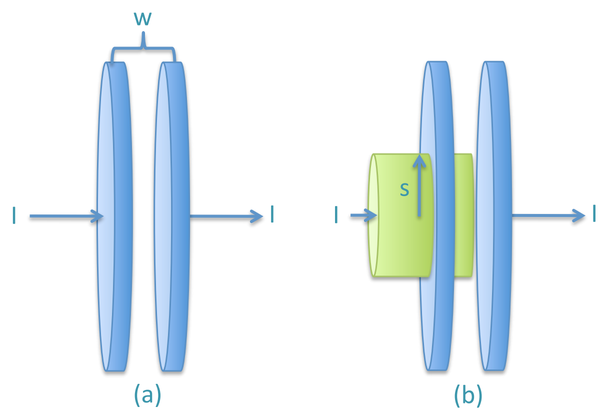
Friday Autumn term week 9, 16h00.

Question 1: Displacement currents 1



A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width w << a, forms a parrell-plate capacitor, as shown in the figure above. Find the magnetic field in the gap, at a distance s < a from the axis. [10%]

Question 2: Displacement currents 2



The preceding problem was an artificial model for the charging capacitor, designed to avoid complications associated with the current spreading out over the surface of the plates. For a more realistic model, imagine thin wires that connect to the centres of the plates (see figure (a)). Again, the current I is constant, the radius of the capacitor is a, and the separation of the plates is w << a. Assume that the current flows out over the plates in a such a way that the surface charge is uniform, at any given time, and is zero at time t = 0.

(a) Find the electric field between the plates, as a function of t. [10%]

(b) Find the displacement current through a circle of radius s in the plane midway between the plates. Using this circle as your "Amperian loop", and the flat surface that spans it, find the magnetic field at a distance s from the axis. [10%]

(c) Repair part b, but this time use the cylindrical surface in figure (b), which is open at the right and extends to the left through the plate and terminates outside the capacitor. Notice that the displacement current through this surface is zero, and there are two contributions to the enclosed current. [25%]

Question 3: Poynting vector

Consider the charging capacitor in Question 1.

(a) Find the electrical and magnetic fields in the gap, as functions of the distance s from the axis and the time t. Assume the charge is zero at t = 0. [5%]

(b) Find the energy density u_{em} and the Poynting vector **S** in the gap. Note especially the direction of **S**. [5%]

(c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector of the appropriate surface. Check that the power input is equal to the rate of increase in energy in the gap. (If you're worried about the fringing fields, do it for a volume of radius $b < a$ will inside the gap.) [25%]

Question 4: Electromagnetic waves, introduction

Write down the (real) electric and magnetic field for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is (a) traveling in the negative x direction and polarized in the z direction; (b) traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the x,z plane. In each case, sketch the wave, and give the explicit Cartesian component of the vectors k and n. [10%]