



$$D_1 = \{(x,y) \mid 0 \leq x \leq \frac{1}{2}, -x+2x^2 \leq y \leq 0\} \quad \text{Type I}$$

$$D_2 = \{(x,y) \mid 0 \leq y \leq 1, y^2 \leq x \leq \frac{1}{4}(1+\sqrt{1+8y})\} \quad \text{Type 2.}$$

$$\text{Hence } D = \{(x,y) \mid 0 \leq x \leq \frac{1}{2}, -x+2x^2 \leq y \leq 0\} \cup \{(x,y) \mid 0 \leq y \leq 1, y^2 \leq x \leq \frac{1}{4}(1+\sqrt{1+8y})\}$$

(v) $I = \iint_D \rho(x,y) dA.$

$$= \int_{x=0}^{\frac{1}{2}} \int_{y=-x+2x^2}^0 x dy dx + \int_{y=0}^1 \int_{x=y^2}^{\frac{1}{4}(1+\sqrt{1+8y})} x dx dy.$$

$$= \int_0^1 [xy]_{y=-x+2x^2}^0 dx + \int_0^1 \left[\frac{1}{2}x^2 \right]_{y^2}^{\frac{1}{4}(1+\sqrt{1+8y})} dy.$$

$$= \int_0^1 [x^2 - 2x^3] dx + \int_0^1 \left[\frac{1}{16} + \frac{1}{16}\sqrt{1+8y} + \frac{y}{4} - \frac{1}{2}y^4 \right] dy$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^4 \right]_0^1 + \left[\frac{1}{16} \left(-\frac{8y^5}{5} + 2y^2 + y + \frac{1}{12}(8y+1)^{3/2} \right) \right]_0^1$$

$$= \frac{1}{3} - \frac{1}{2} - 0 + \left[\left\{ \frac{1}{16} \left(-\frac{8}{5} + 2 + 1 + \frac{1}{12}(9)^{3/2} \right) \right\} - 0 \right]$$

$$I = \frac{59}{960}$$

\therefore The mass of the lamina is $\frac{59}{960}$ units.