Question

Consider the action $S = \frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$. *Vary the potential according to* $A_\mu \rightarrow A_\mu + \partial_\mu\Phi$ *where* Φ *is a scalar field. Determine the variation in the action.*

Solution

$$\delta S = \frac{1}{4}\int d^4x(\delta F_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}\delta F^{\mu\nu}) \tag{1}$$

$$\begin{aligned}\delta F_{\mu\nu}F^{\mu\nu} &= \delta(\partial_\mu A_\nu - \partial_\nu A_\mu)F^{\mu\nu} \\ &= (\partial_\mu\partial_\nu\Phi - \partial_\nu\partial_\mu\Phi)F^{\mu\nu} \\ &= 0\end{aligned}$$

Similarly $F_{\mu\nu}\delta F^{\mu\nu} = 0$. Therefore $\delta S = 0$.