Question

February 17, 2017

1 ……

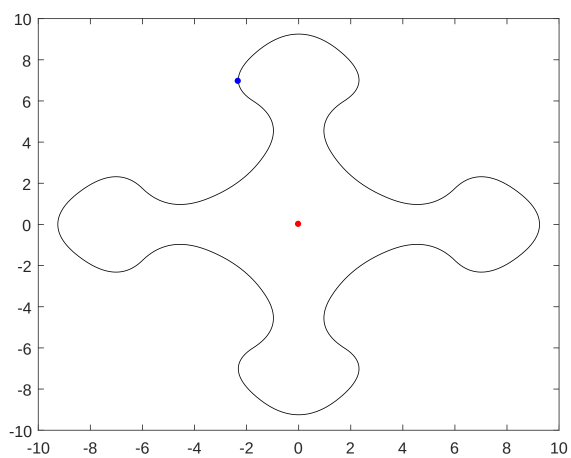
The winding number is the number of revolution made around a point p while travelling once around a close curve $C_{(t)}$. That translated in a mathematical language means:

$$\omega(p, C) = \frac{1}{2\pi} \int_a^b d\theta_{(t)} \tag{1}$$

If we consider the axis centred on p we can re-write the above equation as:

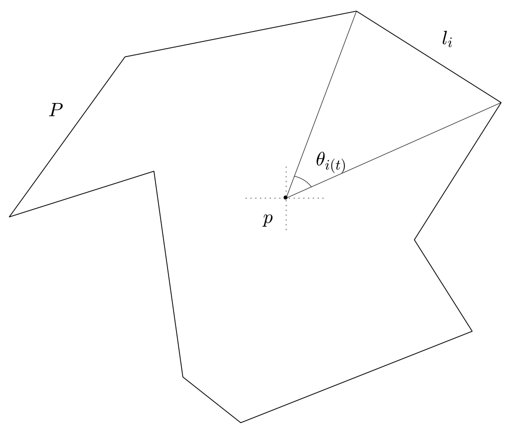
$$\omega(p, C) = \frac{1}{2\pi} \int_a^b \frac{d\theta_{(t)}}{dt} dt = \frac{1}{2\pi} \int_a^b \frac{d}{dt}[atan(y_{(t)}/x_{(t)})]dt \tag{2}$$

this equation shows that there is a kind of singularity if the point p is on the curve C because for a certain t we will find $atan(0/0)$



So considering the figure $\omega(p, C)$ is defined for the red point but not for the blu one. Now if we consider a polygon(a close polygonal chain) it is possible to find the parametrized formulation of the polygon and with a quite hard integration (Ref 'the point in polygon problem for arbitrary polygon ' kay Hormann) the equation Eq. 2 become:

$$\omega(p, P) = \frac{1}{2\pi} \sum_{n=1}^{N_p} \theta_i \tag{3}$$



Now if I consider a point p that lies on an edge l_i of the polygon the Eq. 3 is not undefined because when I meet the edge l_i I find $\theta_i = \pi$. In this discrete situation θ_i looks to be undefined only if the point p lies on a vertex of the polygon and so now the winding number is defined on every edge (=1). I think I get lost during the integration passage and my question is this... is the integration process valid when a point lies on an edge of the polygon?.... if yes it means that it have in some way trasported the singularity only in the case a point lies on a vertex.... if no it means that the Eq. 3 is not written in the right way