Radio frequency measurement of transmission line properties

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1 Aim

To introduce you to the basic concepts of electromagnetic wave transmission along transmission lines.

2 Objectives

- To measure the phase velocity of electromagnetic waves along a particular coaxial-cable
- To measure the characterisitc impedance of the coaxial cable
- To compare the measured values of phase velocity and characteristic impedance with their theoretical values
- To find the inductance of a coil connected as an impedance at the end of a length of the cable

3 Introduction

The aim of this experiment is to introduce you to the propogation of electromagnetic waves along a transmission line. In this particular case the transmission line is a coaxial cable, but the principal holds for other transmission lines. The waves along the transmission line behave much like free-space eletromagnetic waves, but are guided along the line and will experience different impedance (Z_0) and phase velocity (ν_p) to their free-space couterparts. The frequency f, phase velocity (ν_p) and wavelength λ are related in the usual way $\nu_p = f\lambda$.

4 Theory

The basic equation for the voltage V of a wave travelling in the positive x direction along a transmission line is given by the standard equation for a travelling wave

$$V(x) = A \exp j(\omega t - \beta x) \tag{1}$$

where $\omega = 2\pi f$ and $\beta = 2\pi/\lambda$ with λ the wavelength in the transmission line.

For the wave travelling in the positive x direction we can relate the voltage at a given point in the line to the current at that point $I(x)$ through the characteristic impedance of the line Z_0 by:

$$V(x) = Z_0 I(x) \tag{2}$$

and simlarly for the wave travelling in the negative x direction

$$V(x) = -Z_0 I(x) \tag{3}$$

with a change of sign as the current will be flowing in the opposite direction along the line.

If the wave is reflected from the far end of the line then we will have a reflected wave of the form:

$$V(x) = B \exp j(\omega t + \beta x) \tag{4}$$

and the general equation of the wave in the transmission line will be

$$V(x) = A \exp j(\omega t - \beta x) + B \exp j(\omega t + \beta x) \tag{5}$$

$$I(x) = \frac{A}{Z_0} \exp j(\omega t - \beta x) - \frac{B}{Z_0} \exp j(\omega t + \beta x) \tag{6}$$

with the amplitude and phase of the reflected wave depending on the impedance at the end of the line.

Now consider the impedance at the end of the line. For convenience we will take the end of the line to be at $x = 0$ and to have an impedance Z_T. At this point we will have

$$Z_T = \frac{V(0)}{I(0)} \tag{7}$$

so we can substitute in from equations 5, and 6 with $x = 0$ to obtain

$$Z_T = \frac{V(0)}{I(0)} = Z_0 \frac{A + B}{A - B} \tag{8}$$

which can easily be rearranged to give B in terms of A:

$$B = \frac{Z_T - Z_0}{Z_T + Z_0} A \tag{9}$$

We can now calculate the impedance seen at the input end of the line by subsituting this value for B back into our original equations 5, 6. After a small amount of algebra we can see that the input impedance $Z(x)$ at a distance x from the termination of imepdance Z_T is given by:

$$Z(x) = Z_0 \frac{(Z_T + Z_0)\exp(-j\beta x) + (Z_T - Z_0)\exp(j\beta x)}{(Z_T + Z_0)\exp(-j\beta x) - (Z_T - Z_0)\exp(j\beta x)} \tag{10}$$

or even simpler

$$Z(x) = Z_0 \frac{Z_T - jZ_0 \tan\beta x}{Z_0 - jZ_T \tan\beta x} \tag{11}$$

5 Measurement Theory

5.1 Terminated transmission line

Waves launched into the transmission line will travel along it and be reflected from the end of the line and return to the input. The phase of the returned signal will depend on the length of the cable, the wavelength of the wave in the cable and the phase of the reflection at the far end. The signal at the input end of the cable will consist of the superposition of the input signal and signal reflected from the far end of the cable. If these two signals are in phase with each other then the amplitude of the signal at the input is maximised and we see a resonance. Equivalently we could describe this situation as the system having a high input impedance.

For a transmission line of characterisitic impedance Z_0 terminated at $x = 0$ by an impedance of Z_t, the general expression for the impedance Z_i seen at the input end ($x = -l$) is given by equation 11 and is:

$$Z_i = Z_0 \frac{Z_t + jZ_0 \tan(\beta l)}{Z_0 + jZ_t \tan(\beta l)} \tag{12}$$

where $\beta = 2\pi/\lambda$, with λ the wavelength of the electromagnetic wave in the cable.

When the termination is a short-circuit $Z_t = 0$ and the equation becomes

$$Z_i = jZ_0 \tan(\beta l) \tag{13}$$

which shows that the input impedance Z_i goes to high, theoretically infinite, values at certain well defined resonant frequencies given by:

$$f(m) = \frac{(2m+1)\nu_p}{4l} \tag{14}$$

When the termination is open-circuit $Z_t = \infty$ we get

$$Z_i = Z_0 \frac{Z_t + jZ_0 \tan(\beta l)}{Z_0 + jZ_t \tan(\beta l)} \tag{15}$$

which again has the input impedance Z_i going to high, theoretically infinite, values at a set of resonant frequencies, this time given by:

$$f(n) = \frac{n\nu_p}{2l} \tag{16}$$

5.2 Capacitor in parallel with terminated transmission line

In later parts of the experiment we connect capacitors across the input of a line terminated by a short-circuit. In this case the system is slightly more complicated and the resonance occurs when the parallel combination of capacitor and cable assembly has a maximum impedance. The impedance of the capacitor of value C is given by:

$$Z_c = \frac{1}{j\omega \mathrm{C}} \tag{17}$$

and we already have the impedance of the shorted line (equation 13). These combine in parallel to give an total input impedance of

$$Z_{combined} = \frac{jZ_0 \tan(\beta l)}{1 - \omega \mathrm{C} Z_0 \tan(\beta l)} \tag{18}$$

This will have a maximum when

$$\omega \mathrm{C} Z_0 \tan(\beta l) = 1 \tag{19}$$

For simplicity we can use the following substitution:

$$\chi = \frac{1}{\omega \tan(\beta l)} = \frac{1}{2\pi f \tan(2\pi f l/\nu_p)} \tag{20}$$

and the resonance condition (equation 19) can be rewritten as

$$\chi = Z_0 C \tag{21}$$

In practice the capacity C consists of the capacity we have added to the circuit C_x plus some stray capacity $C_s$due to the measurement oscilloscope and its associated wiring. Equation 21 therefore becomes

$$\chi = Z_0 C_x + Z_0 C_s \tag{22}$$

Evaluating the function χ at each resonant frequency corresponding to a value of C_x and ploting them against each other allows you to calculate the characteristic impedance and the stray capacity.

5.3 Transmission line terminated by and inductor

We now consider the more general case of a transmission line terminated by an inductor of value L, and with a with a capacitor C across its input. The situation is similar to the previous one but with the impedance of the line now given by equation 12, with Z_t, the impedance of the inductor given by

$$Z_t = j\omega L \tag{23}$$

We can again look at the resonance condition and rearrange the equation to find the value of the inductor

$$L = \frac{Z_0 \left(1 - 2\pi f C Z_0 \tan \beta l\right)}{2\pi f \left(2\pi f C Z_0 + \tan \beta l\right)} \tag{24}$$

6 Equipment

The arrangement of the equipment used in this experiment is shown in figure 1. The radio-frequency signal generator is used over the frequency range of around 4 MHz to 80 MHz. The basic idea of the measurement is that the resistor and system under test form a voltage divider, where Z_{sys} is the impedance of the system under test. As shown in the diagram, an oscilloscope looks at the size of the signals before and after the resistor (ch1 and ch2). Off-resonance Z_{sys} will be low compared to the input resistor and so the signal seen on channel 2 will be small, however when the system is on resonance its impedance Z_{sys} becomes large and hence the signal on channel 2 will be at its maximum.

The capacitors are manufactured to an accuracy of $\pm 1\,\% \pm 1\,\mathrm{pF}$.

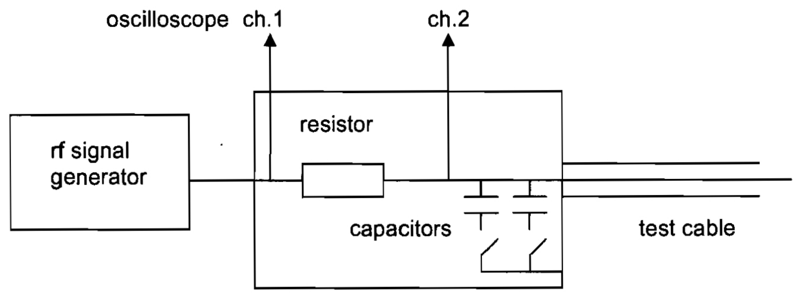

Figure 1: A sketch of the equipment

7 Measurements

7.1 Measurement of phase velocity

Use the arrangement of equipment in the figure to measure the resonant frequencies of a 10m length of the test coaxial cable for both open- and short-circuited conditions. Measure the actual length of the cable. Think about exactly where the end of the cable is on the connector. Plot the resonant frequencies against the integers m and n and find the cable phase velocity.

7.2 Measurement of characteristic impedance

Using the same measurement arrangement, but with a short-circuited cable of about 2m long, switch different capacitor values across the cable input and measure the corresponding lowest resonant frequency for each capacitor value. You can add the capacitors in parallel to get a good range of values. Calculate the values of function χ (Section 5.2), and plot these against the capacitor values. From this graph you should be able to calculate the values of the characterisic impedance and the stray capacity.

7.3 Theoretical values

Measure the dimensions of the sample piece of partially deconstructed cable. Find the theoretical expressions for the phase velocity and the characteristic impedance and use them to calculate the values for your cable.

The dielectric in the cable is polythene.

You will need to borrow the calipers from the lab technician for this measurement.

7.4 Effect of stray capacity

In our initial measurement of the phase velocity in the cable we took the resonant condition given in equation 12. This assumed a "perfect" set up with no stray capacity at the input to the cable. We now know that there is a small stray capacity.

What effect will this stray capacity have on the calculated value of the phase velocity in the cable?

Does the new value of the phase velocity significantly affect the calculated value of the stray capacity?

See appendix A for more details.

7.5 Inductor

Set up the experiment using the 0.5 m length of cable terminated with the small coil. Make measurements of the the lowest resonant frequency of this system for a variety of capacitor values. Use only a few of the smallest values of capacitors. Using your previously calculated values for the characteristic impedance, phase velocity and stray capacity, find the inductance of the coil. See section 5.3 for the details of the calculation.

A Effect of stray capacity

In general the equation for the resonance condition of the short circuited line is with a capacitor across the input is given by equation 19:

$$\omega \mathrm{C} Z_0 \tan(\beta l) = 1 \tag{25}$$

or equivalently:

$$\omega Z_0 C_s = \frac{1}{\tan \frac{\omega l}{v_p}} \tag{26}$$

If we assume that C_s is small ($\omega Z_0 C_s \ll 1$), then the resonance condition can be described as:

$$\frac{2\pi f l}{v_p} = (2n+1)\frac{\pi}{2} - \epsilon \tag{27}$$

This is very similar to the simpler equation we obtained with no capacitor present of

$$\frac{2\pi f l}{v_p} = (2n+1)\frac{\pi}{2} \tag{28}$$

Substituting equation 27 into equation 26 and remembering that:

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \tag{29}$$

we can expand the right hand side of equation 26 with $A = (2n+1)\frac{\pi}{2}$ and $B = \epsilon$ to get:

$$\epsilon \approx \tan \epsilon = 2\pi f Z_0 C_s \tag{30}$$

so at the resonances the equation for resonance is now:

$$\frac{2\pi f l}{v_p} = (2n+1)\frac{\pi}{2} - 2\pi f Z_0 C_s \tag{31}$$

So equation 28 simplifies to

$$f = \frac{(2n+1)}{4}\frac{\nu_p}{l} \tag{32}$$

and equation 31 simplifies to

$$f = \frac{(2n+1)}{4}\frac{1}{\frac{l}{\nu_p} + Z_0 C_s} \tag{33}$$

Thus the presence of C_s modifies the interpretation of the gradient of the graph and will give us a different value of ν_p.