Spin waves in classical gases

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A criterion for the propagation of weakly-damped spin waves is obtained on the basis of a detailed analysis of the quantum collision integral for particles in a paramagnetic polarized Boltzmann gas. It is shown that according to this criterion spin waves can propagate in classical (nonquantum) gases at temperatures close to room temperature. © *1998 American Institute of Physics.*
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In the absence of external electric and magnetic fields, the only propagating collective mode in gases that have Boltzmann statistics is a sound wave. Silin has pointed out the possibility of collective spin oscillations in a paramagnetic gas,[1] Aronov has examined spin-wave oscillations of an electron gas in semiconductors,[2] and Bashkin has predicted the existence of weakly damped spin waves in spin-polarized Boltzmann gases.[3] In Ref. 3 the criterion for the existence of a new collective mode in a spin-polarized gas is reduced to the requirement that the gas be a ''quantum'' gas: The average de Broglie wavelength must be much greater than the size of an atom (scattering amplitude). Despite the large number of works devoted to the dynamics and kinetics of spin-polarized gases (see, for example, Ref. 4 and the references cited therein), this criterion has never been questioned, even though it was based on purely qualitative considerations.[5] It imposes a quite strict constraint on the gas temperature, since most gases condense long before the indicated criterion is first satisfied. For this reason the only appropriate objects were considered to spin-polarized hydrogen $H\uparrow$ and $^3He\uparrow$. The existence of spin waves in these two gases has been confirmed experimentally.[6,7] We underscore that in this case the spin is nuclear spin.

In the present letter a criterion for the propagation of spin waves in a polarized Boltzmann gas is obtained on the basis of a detailed investigation of the collision integral. It is shown that the main quantity governing the propagation of spin waves in Boltzmann paramagnetic gases is the real part of the zero-angle exchange-scattering amplitude. It happens that in ordinary gases, where the scattering of atoms is of a quasiclassical character and sharply anisotropic, the situation is more favorable than in ''quantum'' gases, where scattering is essentially isotropic (*s* scattering).[8] The criterion obtained is different from that adopted in the literature.[3–6] It greatly expands the group of paramagnetic gases where weakly damped spin waves can propagate at temperatures close to room temperature. An example are alkali-metal vapors (Na, Cs, Rb), where a high degree of polarization of the electron spin can be obtained.[9]

Let us consider a paramagnetic gas with externally induced spin polarization (whose vector **M** is directed along the z axis). In this case the Wigner matrix has the form

$$f_{\alpha\alpha'}(p)=f^{(0)}(p)[(\delta_{\alpha\alpha'}+M\sigma_{\alpha\alpha'})+\phi_{\alpha\alpha'}(p)].$$

Here $f^{(0)}(p)$ is the equilibrium Maxwell momentum distribution function (as is the convention in the kinetic theory of gases, the function is normalized to the particle density $n=\int dp f^{(0)}(p)$), $\sigma_{\alpha\alpha'}$ is a vector of Pauli matrices, and $\phi_{\alpha\alpha'}(p)$ is a small perturbation of the Wigner function. It is convenient to decompose the function $\phi_{\alpha\alpha'}(p)$ into scalar (φ) and vector (μ) components:

$$\phi_{\alpha\alpha'}(p)=\varphi(p)\delta_{\alpha\alpha'}+\mu(p)\sigma_{\alpha\alpha'}. \tag{1}$$

The present work is concern with the dynamics of the transverse (with respect to the z axis) component of the magnetic polarization of a gas. It is convenient to combine the quantities μ_x and μ_y as follows: $\mu_\pm=\mu_x\pm i\mu_y$. The kinetic equation for the space–time Fourier components of $\mu_\pm$ has the form

$$[-i\omega\mu_\pm(\omega,k,p)+ikv\mu_\pm(\omega,k,p)]=J_\pm(\mu_\pm). \tag{2}$$

Here ω is the frequency, k is the wave vector, and $J_\pm$ is the collision integral.

The explicit form of the collision integral can be obtained with the aid of Refs. 10 and 11. In view of its complexity, we present an abbreviated form and describe in detail only the term L_I that plays the main role in what follows:

$$J_\pm=Q_R\pm i|M|Q_I\mp i|M|L_I. \tag{3}$$

Here Q_R and Q_I are integral operators which are quadratic in the T matrix (scattering amplitude) and whose structure has the same character as in the conventional Boltzmann equation. The eigenvalues of the operators Q_R and Q_I are of the same order of magnitude as $\nu_s\simeq n\bar{v}\sigma$, where σ is the gas kinetic collision cross section and $\bar{v}$ is the average thermal velocity of the atoms.

The structure of the operator L_I is substantially different:

$$L_I(\mu_\pm)=16\pi^3\hbar^2\int dp_1 f^{(0)}(p_1)$$
$$\times\mathrm{Re}\left\{T_{\mathrm{ex}}\left(\frac{p-p_1}{2},\frac{p-p_1}{2}\right)[\mu_\pm(p)-\mu_\pm(p_1)]\right\}. \tag{4}$$

Here T_{ex} is the T matrix of spin-exchange scattering $(\uparrow\downarrow\rightarrow\downarrow\uparrow)$:

$$T_{\mathrm{ex}}(P,P')=2\theta(P,P')+\theta(-P,P')-t(-P,P'),$$

P and P' are the relative momenta of the colliding particles, scaled to the reduced mass of the atoms $(m/2)$, while t and θ are related with the total scattering matrix (for identical fermions) as follows:

$$T_{\alpha\beta\mu\nu}(P,P')=\hat{A}[t(P,P')\delta_{\alpha\mu}\delta_{\beta\nu}+\theta(P,P')\sigma_{\alpha\mu}\sigma_{\beta\nu}]$$
$$=t(P,P')\delta_{\alpha\mu}\delta_{\beta\nu}-t(-P,P')\delta_{\alpha\nu}\delta_{\beta\mu}+\theta(P,P')\sigma_{\alpha\mu}\sigma_{\beta\nu}-\theta$$
$$\times(-P,P')\sigma_{\alpha\nu}\sigma_{\beta\mu}. \tag{5}$$

Here $\hat{A}$ denotes antisymmetrization with respect to an interchange of the particles (incident and scattered). The product of Pauli matrices is a scalar product: $\Sigma_j \sigma^{(j)}_{\alpha\mu}\sigma^{(j)}_{\beta\nu}$. It is easy to show that formula (5) describes collisions which conserve the total spin. This is a good approximation not only in the case of nuclear spin (H, ^{3}He) but also for atoms with electronic spin (alkali-metal vapors).[9]

All three operators Q_R, Q_I, and L_I are Hermitian in the space L of the functions $\mu(p)$ with the scalar product conventionally used in the kinetic theory of gases,[12] so that the operators J_+ and J_- are Hermitian conjugates of each other. The function $|1\rangle = \mu_\pm \equiv 1$ is an eigenfunction of the operators $J_\pm$ with zero eigenvalue, which is a consequence of the conservation of the total spin in collisions (see Eq. (5)). Moreover, the zero eigenvalue of $J_\pm$ is nondegenerate, since there are no other conservation laws in the ''$\mu_\pm$ subspace.'' The conservation laws for momentum, energy, and particle number refer to the diagonal elements of the Wigner matrix.

Mathematically, Eq. (2) is an eigenvalue problem for the operators $J_\pm - ik\hat{v}$ ($\hat{v}$ denotes multiplication by the velocity treated as an operator). For small k ($|kv| \ll \nu$, where ν is the absolute magnitude of the characteristic eigenvalue of the operator $J_\pm$), this problem can be solved by perturbation theory with respect to the operator $ik\hat{v}$, i.e., in the hydrodynamic approximation.[12] The first-order perturbation theory correction equals zero because of the isotropy of the equilibrium velocity distribution function.

In the present case the second-order correction can be written as[12]

$$-i\omega_\pm = k^2\langle 1|\hat{v}J_\pm^{-1}\hat{v}|1\rangle. \tag{6}$$

Here $J_\pm^{-1}$ is the inverse of the operator $J_\pm$ on the subspace $(1-\hat{P})L$ ($\hat{P}=|1\rangle\langle 1|$): The operator $J_\pm$ is noninvertible in the initial space L, since it has a zero eigenvalue.

Let us consider first an unpolarized gas ($M=0$). In this case $J_+ = J_- = Q_R$. The eigenvalues of Q_R (and Q_R^{-1}) are real, and therefore ω is purely imaginary. Therefore in this case we have simply spin diffusion. As usual,[12] in this situation it is possible only to estimate the corresponding diffusion coefficient:

$$D_s = \langle 1|\hat{v}J_R^{-1}\hat{v}|1\rangle. \tag{7}$$

In order of magnitude $D_s \sim \overline{v^2}/3\nu_s$, where $1/\nu_s$ is the characteristic eigenvalue of the operator Q_R^{-1}. The spin diffusion coefficient D_s in the general case is different from other transport coefficients, since the corresponding effective collision frequencies are different.

The situation is substantially different in a polarized gas. Let $|M| \sim 1$ (the requirement that the density matrix be positive-definite gives the condition $0 \leqslant |M| \leqslant 1$, and for optical polarization of a paramagnetic gas a value of $|M|$close to 1 can be achived in a straightforward way). Now, the operators $J_\pm$ essentially reduce to the third term on the right-hand side of Eq. (5), while the first two operators can be treated as a small correction.

To show this we shall estimate the real part $\mathrm{Re}[T_{ex}(0)]$ of the zero-angle exchange-scattering amplitude. It follows from the general formula for the T matrix[13] that the Born term makes the main contribution to the zero-angle scattering matrix for fast atoms, since the oscillations of the factor $\exp(ikx)$, which describes the incident wave ($k = p/\hbar$ is the

wave vector of the atom), are completely cancelled out in it. At room temperature (and actually at lower temperatures also) the relation $|k|a_0 \gg 1$, where a_0 is the effective interaction radius, holds by a large margin for electron spin exchange. This last inequality means that the atoms are fast. Thus the operator L_I, or, more accurately, its characteristic eigenvalue ν_{ex}, is proportional to the Born amplitude for scattering by zero angle:

$$A(0) = -\frac{m}{4\pi\hbar^2}\int U d^3x \simeq -\frac{m}{4\pi\hbar^2}|U|a_0^3$$

(U is the exchange interaction potential).

The ratio ν_{ex}/ν_s can now be easily estimated. It equals in order of magnitude

$$\frac{\nu_{ex}}{\nu_s} \sim |M|\frac{|U|a_0}{\hbar\overline{v}}. \tag{8}$$

We note that the right-hand side of this relation is, to within the factor $|M|$, the so-called ''Born parameter,''[8] which is usually large in classical (nonquantum) gases. A numerical estimate of the Born parameter for cesium atoms will be presented below as an example.

So, to a first approximation in the (inverse) Born parameter Eq. (2) can be represented in the form

$$(kv-\omega)\mu_\pm(p) = \mp 16|M|\pi^3\hbar^2\int dp_1 f^{(0)}(p_1)\mathrm{Re}\ T_{ex}(0)[\mu_\pm(p)-\mu_\pm(p_1)]. \tag{9}$$

This equation can once again be studied in perturbation theory, just as we did earlier. Here, however, a different method will be used for estimates. To simplify the problem let $T_{ex} =$ const, i.e., T_{ex} is independent of energy. Then

$$(kv-\omega)\mu_\pm(p) = \mp\nu_{ex}\left[\mu_\pm(p) - \frac{1}{n}\int dp_1 f^{(0)}(p_1)\mu_\pm(p_1)\right]$$

(but now $\nu_{ex} = 16|M|\pi^3\hbar^2 n\mathrm{Re}T_{ex}$).

The following dispersion relation is obtained in the hydrodynamic approximation ($|k\overline{v}/\nu_{ex}| \ll 1$, $|\omega/\nu_{ex}| \ll 1$):

$$\omega = \pm\frac{k^2\overline{v^2}}{3\nu_{ex}}. \tag{10}$$

This equation describes an undamped spin-polarization wave. The diffusion damping Γ of the wave is an effect of the next order in the inverse Born parameter ν_s/ν_{ex}. The quantity Γ can be estimated as

$$\Gamma \sim \frac{k^2\overline{v^2}}{\nu_{ex}}\frac{\nu_s}{\nu_{ex}} \ll \omega. \tag{11}$$

Therefore weakly damped spin waves, whose frequency and damping are estimated by Eqs. (10) and (11), can propagate in a paramagnetic polarized Boltzmann gas. We note that the diffusion damping Γ (11) of spin waves in a polarized gas is $(\nu_s/\nu_{ex})^2$ times smaller than the corresponding quantity in an unpolarized gas (7).

As an example, let us consider the parameters of spin waves in vapor of polarized cesium, for which the required experimental data are available. According to Ref. 14, electronic spin polarization in cesium vapors is conserved to a high degree of accuracy: The ratio of the cross sections for collisions with spin nonconservation and spin exchange is of the order of 1%. The quantity $a_0 \sim 10^{-7}$ cm.[14,15] We take as an estimate of $|U|$ a values of twice the binding energy of the Cs_2 molecule: $|U| \sim 1$ eV.[16] Then the Born parameter at an average thermal velocity of cesium atoms $\bar{v} = 2 \times 10^4$ cm/s is of the order of 10^3. This large value of the Born parameter is intimately connected with the pronounced anisotropy of the scattering indicatrix of fast particles.[8] We note that the temperature dependence of this parameter derives mainly from the dependence $\bar{v} \sim \sqrt{T}$. Therefore the Born parameter approaches 1 only at temperatures less than 10^{-4}–10^{-3} K.

According to Eqs. (10) and (11), the frequency-to-damping ratio of a spin wave is

$$\frac{\omega}{\Gamma} \sim 10^3 |M|.$$

It follows hence that narrowing increases with $|M|$. It should be kept in mind that, generally speaking, this estimate may need to be refined by taking into account the actual weak spin nonconservation.

In summary, the condition for spin-wave propagation in a polarized Boltzmann gas is determined by two factors: spin conservation, i.e., the slowness of spin destruction in collisions, and a sharp anisotropy of the scattering phase function for atoms at room temperature. As is well known, in this case the scattering occurs mainly by small angles near $(|k|a_0)^{-1}$ $(|k| = m\bar{v}/\hbar)$. For cesium at room temperature the corresponding range of angles is $\simeq 10^{-3}$.

To observe spin waves in alkali-metal vapors, besides the conventional magnetic-resonance method,[17] it is of interest to employ the method of scattering of light near the resonance D lines of the atoms. As was shown in Ref. 18, the polarizability of an atom near resonance lines has a large antisymmetric component, which makes it possible to observe the kinetics of spin fluctuations. In the presence of spin waves the spectrum of electron spin polarization fluctuations should consist of a well-resolved doublet with narrow components.

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