

$$S = \sum \frac{1}{(1+y)^n}$$

$$S = 1 + \cancel{\frac{1}{r}} + \cancel{\frac{1}{r^2}} + \cancel{\frac{1}{r^3}} + \dots + \cancel{\frac{1}{r^n}}$$

$$\frac{S}{r} = \cancel{\frac{1}{r}} + \cancel{\frac{1}{r^2}} + \cancel{\frac{1}{r^3}} + \dots + \frac{1}{r^{n+1}}$$

$$S(1 - \frac{1}{r}) = 1 - \frac{1}{r^{n+1}}$$

$$\therefore S = \frac{1 + \frac{1}{r^{n+1}}}{(1 - \frac{1}{r})} = \frac{1 - \frac{1}{r^{n+1}}}{(\frac{r}{1} - 1)} = \frac{1 - \frac{1}{r^{n+1}}}{\frac{r-1}{r}} = \frac{r(1 - \frac{1}{r^{n+1}})}{r-1}$$

Replacing r with $(1+y)$, we get

$$= \frac{(1+y)(1 - \frac{1}{(1+y)^{n+1}})}{\cancel{1+y} - 1} = \frac{(1+y) - \frac{1}{(1+y)^n}}{y}$$