The attempt at a solution

Let's first show that for any sequence (s_n) and any number M, if $s_n > M$ for all $n \in \mathbb{N}$, then $\limsup s_n \geq M$. Since $s_n > M$ for all $n \in \mathbb{N}$, then it follows that $s_n > M$ for all $n > N$ for some $N \in \mathbb{N}$. Now, let's denote $v_N = \sup\{s_n : n > N\}$. Since v_N is the least upper bound for s_n when $n > N$, then it follows that we have $v_N \geq s_n > M$ for $n > N$. And note that we have the property $v_N \geq v_{N+1} \geq \ldots$ and so (v_N) is monotonically decreasing and since (s_n) is bounded, by the monotone convergence theorem, $\lim_{N\to\infty} v_N$ exists and is real. Moreover, this is precisely the definition of the limit superior and hence, we have that $\lim_{N\to\infty} v_N = \lim_{N\to\infty} \sup\{s_n : n > N\} = \limsup s_n$. Thus, we have $\limsup s_n > M$ as desired.

For contradiction, suppose that x is the limit of some subsequence (s_{n_k}) of the sequence (s_n). Let's first denote $v = \limsup s_n$. Note that if $\lim_{k\to\infty} s_{n_k} = x > v$, then there exists some N such that for all $k > N$, we have $s_{n_k} > v$. Now, consider $s_{n_k} > v$ for all $k > N$. Then by the result above, and since removing finite number of terms $1 \leq k \leq N$ does not affect convergence results, it implies $\limsup s_{n_k} > v = \limsup s_n$ — contradiction, since $\{s_{n_k} : k > N\} \subseteq \{s_n : n > N\}$, it implies $\sup\{s_{n_k} : k > N\} \leq \sup\{s_n : n > N\}$ and thus $\lim_{N\to\infty} \sup\{s_{n_k} : k > N\} \leq \lim_{N\to\infty} \sup\{s_n : n > N\}$. ∎