Objectives

The goal of this analysis is to estimate the impact of improving the percentage of balls that are returned, while holding everything else constant. That is, if I improve my ball return percentage by 5 points, how will my odds of winning a point, game, set, and match increase. We will start with points and work up through games and sets to matches.

Variables

Givens

Name	Use
A	Player A. Most of the analysis is from A's perspective.
B	Player B. The other player, A's opponent.
$PS1_A$	Average probability of A getting the first serve in.
$PR1_B$	Average probability of B getting As first serve back.
$PS2_A$	Average probability of A getting the second serve in.
$PR2_B$	Average probability of B getting As second serve back.
PGS_A	Average probability of A making a successful groundstroke.
PGS_B	Average probability of B making a successful groundstroke.

Calculations

Name	Use
PPA_A	Probability of A winning a point started by A.
PPA_B	Probability of B winning a point started by A based on PR_A & PR_B. This should equal 1-PPA_A.

Groundstrokes & Points

We will start by calculating the odds of A winning a point given the average odds of both A and B making a successful groundstroke. That is, returning the ball each time they have a chance after the serve and return. We will ignore the serve for now.

If PGS_A = and PGS_B are the average probabilities of Players A & B making a successful groundstroke, then the probability of A winning a point started by A, PPA_A, is:

$$\begin{aligned} PPA_A &= PGS_A \times (1 - PGS_B) + PGS_A \times PGS_B \times PGS_A \times (1 - PGS_B) + \cdots \\ &= PGS_A \times (1 - PGS_B) \times ((PGS_B \times PGS_A)^0 + (PGS_B \times PGS_A)^1 + (PGS_B \times PGS_A)^2 + \cdots) \end{aligned}$$

This is the standard geometric series,

$$\sum_{i=0}^{\infty} ar^i = a\frac{1}{1-r}; \ \ |r| < 1$$

where

$$a = PGS_A \times (1 - PGS_B)$$

and

$$r = PGS_B \times PGS_A$$

Therefore,

$$PPA_A = PGS_A \times (1 - PGS_B)\frac{1}{1 - PGS_B \times PGS_A} = \frac{PGS_A \times (1 - PGS_B)}{1 - PGS_B \times PGS_A}$$

As a validity check, let's calculate PPA_B, the odds of Player B winning that same point.

$$PPA_B = (1 - PGS_A) + PGS_A \times PGS_B \times (1 - PGS_A) + (PGS_A \times PGS_B)^2 \times (1 - PGS_A) + \cdots$$

$$= (1 - PGS_A)\frac{1}{1 - PGS_A \times PGS_B} = \frac{1 - PGS_A}{1 - PGS_A \times PGS_B}$$

These two probabilities should add to "1".

$$PPA_A + PPA_B = \frac{PGS_A \times (1 - PGS_B)}{1 - PGS_B \times PGS_A} + \frac{1 - PGS_A}{1 - PGS_A \times PGS_B}$$

$$= \frac{PGS_A - PGS_A \times PGS_B + 1 - PGS_A}{1 - PGS_B \times PGS_A} = \frac{1 - PGS_A \times PGS_B}{1 - PGS_B \times PGS_A} = 1$$

Yea!!! ☺

Now let's see how the numbers come out. In this first example, both players return their groundstrokes 50% of the time on average. Note that the player who starts the point is only half as likely to win it as iw the opponent – 33% to 67%.

50%	PGS_A = Average probability of Player A making a successful groundstroke.				
50%	PGS_B = Average probability of Player B making a successful groundstroke.				
33%	PPA_A = Probability of A winning a point started by A.				
67%	PPA_B = Probability of B winning a point started by A.				
100%	$PPA_A + PPA_B$				

This is illustrated in this table showing the odds of winning the point after each groundstroke. Since A fails to get the ball back 50% of the time, B wins half of the points without hitting the ball. If A gets it back, B then misses half of the time, so A wins 25% of the time (0.5 x 0.5).

Probabilities of Winning Point after each Groundstroke					
#GS	Player B		Player A		A+B
1	50.000%	50.000%	25.000%	25.000%	75.00%
2	12.500%	62.500%	6.250%	31.250%	93.75%
3	3.125%	65.625%	1.563%	32.813%	98.44%
4	0.781%	66.406%	0.391%	33.203%	99.61%
5	0.195%	66.602%	0.098%	33.301%	99.90%
6	0.049%	66.650%	0.024%	33.325%	99.98%
7	0.012%	66.663%	0.006%	33.331%	99.99%
8	0.003%	66.666%	0.002%	33.333%	100.00%
9	0.001%	66.666%	0.000%	33.333%	100.00%
10	0.000%	66.667%	0.000%	33.333%	100.00%

If both players return their groundstrokes 75% of the time, the player to hit first is still at a disadvantage 43% to 57%.

75%	PGS_A = Average probability of Player A making a successful groundstroke.
75%	PGS_B = Average probability of Player B making a successful groundstroke.
43%	PPA_A = Probability of A winning a point started by A.
57%	PPA_B = Probability of B winning a point started by A.
100%	$PPA_A + PPA_B$

Probabilities of Winning Point after each Groundstroke

#GS	Player B		Player A		A+B
1	25.000%	25.000%	18.750%	18.750%	43.75%
2	14.063%	39.063%	10.547%	29.297%	68.36%
3	7.910%	46.973%	5.933%	35.229%	82.20%
4	4.449%	51.422%	3.337%	38.567%	89.99%
5	2.503%	53.925%	1.877%	40.444%	94.37%
6	1.408%	55.333%	1.056%	41.500%	96.83%
7	0.792%	56.125%	0.594%	42.094%	98.22%
8	0.445%	56.570%	0.334%	42.428%	99.00%
9	0.251%	56.821%	0.188%	42.616%	99.44%
10	0.141%	56.962%	0.106%	42.721%	99.68%

Even if the players return balls at 98%, the player to hit first is still at a slight disadvantage.

98%	PGS_A = Average probability of Player A making a successful groundstroke.
98%	PGS_B = Average probability of Player B making a successful groundstroke.
49%	PPA_A = Probability of A winning a point started by A.
51%	PPA_B = Probability of B winning a point started by A.
100%	$PPA_A + PPA_B$

Probabilities of Winning Point after each Groundstroke

#GS	Player B		Player A		A+B
1	2.000%	2.000%	1.960%	1.960%	3.96%
2	1.921%	3.921%	1.882%	3.842%	7.76%
3	1.845%	5.766%	1.808%	5.650%	11.42%
4	1.772%	7.537%	1.736%	7.386%	14.92%
5	1.702%	9.239%	1.667%	9.054%	18.29%
6	1.634%	10.873%	1.601%	10.655%	21.53%
7	1.569%	12.442%	1.538%	12.193%	24.64%
8	1.507%	13.950%	1.477%	13.671%	27.62%
9	1.448%	15.397%	1.419%	15.089%	30.49%
10	1.390%	16.787%	1.362%	16.452%	33.24%