

If A and B are any two sets, their symmetric difference is the set $A + B$ defined as follows: $A + B = (A - B) \cup (B - A)$.

If D is a set, then the power set of D is the set P_D of all the subsets of D. That is, $P_D = \{A : A \subseteq D\}$

1. Prove that there is an identity element with respect to the operation $+$. Let e be the identity element.

$$\begin{aligned} A + e &= A \\ (A - e) \cup (e - A) &= A \end{aligned}$$

As a result there are two cases: $(A - e) = A$ or $(e - A) = A$

$(A - e) = A$ can be interpreted as removing from A all the elements that belong to e to obtain A. It is clear that e must be the \emptyset for this to be true.

Because we determined that e must be the \emptyset , $(e - A) = A$ must not be valid since there is nothing to remove from an empty set.

2. Prove that every subset A of D has an inverse with respect to $+$. Let A^{-1} be the inverse.

$$\begin{aligned} A + A^{-1} &= \emptyset \\ (A - A^{-1}) \cup (A^{-1} - A) &= \emptyset \end{aligned}$$

There are two cases: $A - A^{-1} = \emptyset$ or $A^{-1} - A = \emptyset$

$A - A^{-1} = \emptyset$ can be interpreted as removing from A all the elements that belong to A^{-1} to obtain the empty set. It is clear that $A^{-1} = A$.

$A^{-1} - A = \emptyset$ can be interpreted as removing from A^{-1} all the elements that belong to A to obtain the empty set. Thus $A = A^{-1}$