If a constant external force is exerted on the end of a long slender member in the longitudinal direction, the portion of the member's total mass that will be influenced by the external force will vary with time until information about the force has travelled to the opposite end of the member. If this opposite end of the member is fixed, then the deformation of the member will cease when information about the external force reaches the fixed support and the support exerts an opposite force of equal magnitude upon the entire member, which will behave as a single entity at that time. In a one-dimensional material this information about the external force travels with a velocity $c = \sqrt{\frac{E}{\rho}}$, where E and ρ are the the modulus of elasticity and density of the material, respectively. Newton's 2nd law states that

$$\sum F = ma = m\frac{d^2x}{dt^2},$$

so that $a = \frac{\sum F}{m} = \frac{d^2x}{dt^2}$. Differentiating with respect to time yields

$$\frac{da}{dt} = -\left(\frac{F}{m(t)^2}\right)\frac{dm}{dt} = \frac{d^3x}{dt^3}.$$

If the long slender member has a uniform cross-sectional area, then

$dm = \rho dV = \rho A dx$, so that $\frac{dm}{dt} = \rho A\frac{dx}{dt} = \rho A\sqrt{\frac{E}{\rho}}$. Substituting this result into the previous result yields

$$\frac{da}{dt} = -\frac{F\rho A}{m(t)^2}\sqrt{\frac{E}{\rho}} = \frac{d^3x}{dt^3}.$$

$m(t)$ increases linearly, so that

$$m(t) = m_0 + \frac{dm}{dt}t = \rho A\Delta x + \rho A\sqrt{\frac{E}{\rho}}\,t$$

where m_0 is the effective initial mass of the object and Δx is the length corresponding to this mass, so that

$$\frac{da}{dt} = -\frac{F\rho A}{\left(\rho A\Delta x + \rho A\sqrt{\frac{E}{\rho}}\,t\right)^2}\sqrt{\frac{E}{\rho}} = -\frac{FA\sqrt{\rho E}}{\left(\rho A\Delta x + \rho A\sqrt{\frac{E}{\rho}}\,t\right)^2} = \frac{d^3x}{dt^3}.$$

Integrating once with respect to t yields

$$a(t) = \frac{dx^2}{dt^2} = \frac{F}{A(\Delta x\,\rho + \sqrt{\rho E}\,t)} + C.$$

Since $a(0) = \frac{F}{m_0} = \frac{F}{\rho A\Delta x}$, $\frac{F}{A\Delta x\,\rho} + C = \frac{F}{\rho A\Delta x}$ so that $C = 0$ and

$$a(t) = \frac{F}{A(\Delta x\,\rho + \sqrt{\rho E}\,t)}.$$

Integrating a second time with respect to t yields

$$v(t) = \frac{F\,ln(\Delta x\,\rho + \sqrt{\rho E}\,t)}{A\sqrt{\rho E}} + C.$$

$v(0) = \frac{F\,ln(\Delta x\,\rho)}{A\sqrt{\rho E}} + C = 0$ so that $C = -\frac{F\,ln(\Delta x\,\rho)}{A\sqrt{\rho E}}$ and

$$v(t) = \frac{F\,ln\Big(1 + \frac{t}{\Delta x}\sqrt{\frac{E}{\rho}}\,\Big)}{A\sqrt{\rho E}}.$$

Integrating a third time with respect to t yields

$$x(t) = \frac{F\big(t\sqrt{E} + \Delta x\sqrt{\rho}\big)\big(\ln\Big(1 + \frac{t\sqrt{E}}{\Delta x\sqrt{\rho}}\Big) - 1\big)}{AE\sqrt{\rho}} + C.$$

$x(0) = 0$, so that $-\frac{F\Delta x}{AE} + C = 0$, $C = \frac{F\Delta x}{AE}$ and

$$x(t) = \frac{F\big(\big(t\sqrt{E} + \Delta x\sqrt{\rho}\big)\ln\Big(1 + \frac{t\sqrt{E}}{\Delta x\sqrt{\rho}}\Big) - t\sqrt{E}\,\big)}{AE\sqrt{\rho}}.$$

At time $t = \frac{L}{\sqrt{\frac{E}{\rho}}} = L\sqrt{\frac{\rho}{E}}$ the long slender member will have experienced the maximum possible amount of longitudinal deflection. This substitution yields

$$\frac{F\big((L + \Delta x)\ln\big(\frac{L}{\Delta x} + 1\big)\big)}{AE}.$$

If

$$\frac{FL}{AE} = \frac{F((L + \Delta x)\ln(\frac{L}{\Delta x} + 1))}{AE},$$

then it follows that

$$\ln\Big(\frac{L}{\Delta x} + 1\Big) = \frac{2L}{L + \Delta x}.$$

Given the difficulty of solving the preceeding equation, it is probably best if numeric root finding methods are utilized for this purpose. The results of such an approach indicate that $\Delta x(L)$ can be expressed with a high degree of accuracy as $\Delta x(L) = 0.255L$, so that this value for Δx can be substituted into the time-dependent equation for position, which will the give the position of the end of the rod relative to its initial position at any point in time.