Deriving Electric Field from Electric Potential for a Spherical Shell

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1 Statement of part of the question that I am having trouble with

Use the integral (3.32) to determine the potential V($\vec{x}$) both inside and outside a uniformly charged spherical shell ,with total charge Q and radius R. From V($\vec{x}$) calculate the electric field. Note while V(r) is continuous, $\vec{E(\vec{r})}$ is not due to the charge on the surface.

2 Relevant Equations

Integral 3.32: $V(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{x'})}{|\vec{x}-\vec{x'}|} d^2x'$

3 My work thus far/My question

Ok, so I set up my original integral in spherical coordinates as $\frac{\sigma}{4\pi\varepsilon_0} \int\limits_0^{2\pi} \int\limits_0^{\pi} \frac{R^2}{r} sin\theta d\theta d\phi$ Where R is the radius of the sphere. After integrating I get $\frac{\sigma R^2}{\varepsilon_0 r}$ as the expression of the electric potential. The problem is that when I take the negative gradient in spherical coordinates in order to try to get the radial component of the electric field, I get $\frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r}$ without any indication that there is a discontinuity at the surface. However, I know that from Gauss's Law, the field HAS TO BE discontinuous since there is no charge within the shell. My problem is I am not understanding why my approach is not indicating any discontinuity anywhere.

I also would like to point out that in googling this problem, I came across an approach that already assumes the Electric Field is known and uses $V(\vec{x}) = \int \vec{E} \cdot \vec{dl}$ to show that while the field is discontinuous, the potential is continuous. While this further elaborated to me why V is continuous and I know the theoretical reason as to why this discontinuity over a charged surface exists (I have read and reread that section in my text a dozen times now). I am just trying to figure out how to see that in this problem.