The base of an isosceles triangle is 20 cm and the altitude is increasing at the rate of 1 cm/min. At what rate is the base angle increasing when the area is 100 cm2?

$$A = \frac{1}{2} \times b \times h$$

A = 100 cm^2

B = 20 cm

$$100 = \frac{1}{2} \times 20 \times h$$

Solve for *h*:

$$100 = 10h$$

$$h = 10$$

The angle of the base:

$$\tan\theta = \frac{h}{10}$$

Substitute the value of h which we found in the previous section into the equation, and solve for θ(the base angle):

$$\tan\theta = \frac{h}{10}$$

$$\tan\theta = \frac{10}{10}$$

$$\theta = \tan^{-1}\frac{10}{10}$$

$$\theta = 0.7854 \ \text{rads}$$

Differentiate to find the rate of change:

$$\left(\frac{d\theta}{dt}\right)\sec^2\theta = \frac{1}{10}\left(\frac{dh}{dt}\right)$$

Solve for $\frac{d\theta}{dt}$:

$$\left(\frac{d\theta}{dt}\right)\sec^2\theta = \frac{1}{10}\left(\frac{dh}{dt}\right)$$

$$\frac{dh}{dt} = 1\mathrm{cm}^2/\mathrm{min}$$

$$\left(\frac{d\theta}{dt}\right) = \frac{1}{10}\times\frac{1}{\sec^2\theta}(1)$$

$$\frac{d\theta}{dt} = \frac{1}{10}\times\frac{1}{\sec^2\theta}$$

We must substitute the value of $\theta = 0.7854$ rads to find the rate of change of the base angle:

$$\frac{d\theta}{dt} = \frac{1}{10}\times\frac{1}{\sec^2\theta}$$

$$\frac{d\theta}{dt} = \frac{1}{10}\times\frac{1}{\sec^2(0.7854)}$$

$$\frac{d\theta}{dt} = \frac{1}{20}$$

$$\frac{d\theta}{dt} = 0.05 \text{ radians/min}$$