

THE PLANCK SCALE

By international agreement, the distance or length, L , between two points is defined as the time, t , it takes for light to travel between the points in a vacuum, multiplied by a constant, c :

$$L = ct \tag{1}$$

While c is called the speed of light in a vacuum, it is an arbitrary number that simply sets the units of distance. For example, if t is measured in seconds and you want L in meters, then $c = 3 \times 10^8$ by definition. Physicists and astronomers often work in units where $c = 1$, for example, when t is in years and L in light-years.

In order to measure t we need a clock with an uncertainty, Δt , no larger than t . The time-energy uncertainty principle says that the product of Δt and the uncertainty in a measurement of energy in that time interval, ΔE , can be no less than $\hbar/2$, where $\hbar = h/2\pi$ and $h = 6.63 \times 10^{-34}$ Joule-sec is Planck's constant.

$$\Delta E t \geq \Delta E \Delta t \geq \frac{\hbar}{2} \tag{2}$$

Thus,

$$\Delta E \geq \frac{\hbar}{2t} \geq \frac{\hbar c}{2L} \tag{3}$$

This energy equals the rest energy of a body of mass m ,

$$\Delta E = mc^2 \tag{4}$$

Let L be the radius of a sphere. Within a spherical region of space of radius L we cannot determine, by any measurement, that it contains a mass less than

$$m = \frac{\hbar}{2cL} \quad (5)$$

Now, let us consider the special case where the gravitational potential energy of a spherical body of mass m and radius R equals half its rest energy,

$$\frac{1}{2} mc^2 = \frac{Gm^2}{R} \quad (6)$$

so that,

$$R = \frac{2Gm}{c^2} \quad (7)$$

This is called the *Schwarzschild radius*. According to general relativity, any body of mass m with radius less than R is a black hole. Suppose that $L = R$. Let us call that special case L_{PL} . Then, from (5) and (7),

$$L_{PL} = \left(\frac{\hbar G}{c^3} \right)^{\frac{1}{2}} \quad (8)$$

which is called the *Planck length*, $L_{PL} = 1.6 \times 10^{-35}$ meter. We can see that it represents the smallest length that can be operationally defined, that is, defined in terms of measurements that can be made with clocks and other instruments. If we tried to measure a smaller distance, the time interval would be smaller, the uncertainty in rest energy larger, the uncertainty in mass larger, and the region of space would be experimentally indistinguishable from a black hole. Since nothing inside a black hole can climb outside its gravitational field, we cannot see inside and thus cannot make smaller measurement of distance.

Similarly, we can make no smaller measurement of time than the *Planck time*,

$$t_{PL} = \frac{L_{PL}}{c} = \left(\frac{\hbar G}{c^5} \right)^{\frac{1}{2}} \quad (9)$$

which has the value $t_{PL} = 5.4 \times 10^{-44}$ seconds. Also of some interest are the *Planck mass*,

$$m_{PL} = \frac{\hbar}{cL_{PL}} = \left(\frac{\hbar c}{G} \right)^{\frac{1}{2}} \quad (10)$$

which has a value of 2.2×10^{-8} kilogram, and the Planck energy,

$$E_{PL} = m_{PL} c^2 \left(\frac{\hbar c^5}{G} \right)^{\frac{1}{2}} \quad (11)$$

which has a value of 2.0×10^9 Joules or 1.2×10^{28} electron-volts. These represent the uncertainty in rest mass and rest energy within the space of a Planck sphere or within a time interval equal to the Planck time.