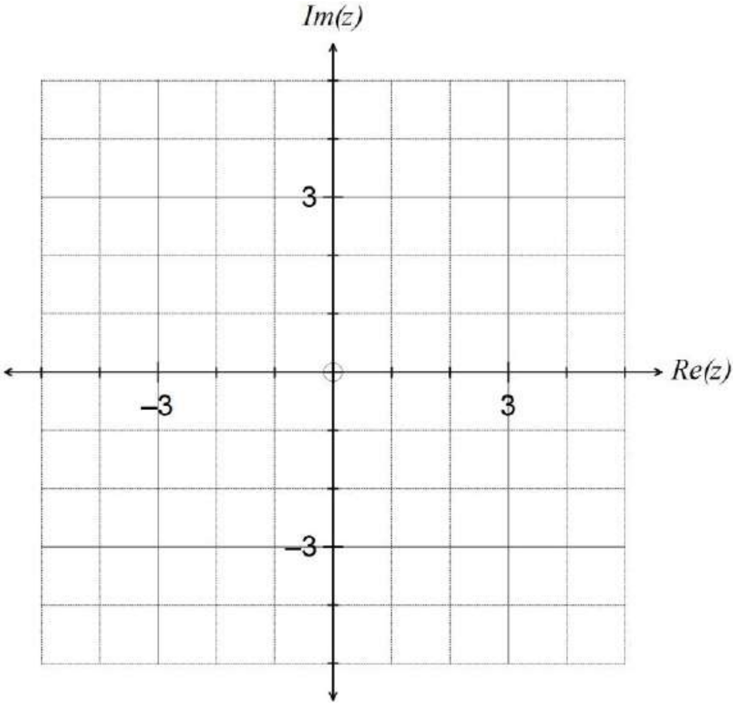
Question 1 **(5 marks)**

(a) Plot and label the complex number $z = -1 - 2i$ on the Argand diagram below. (1 mark)



(b) On the same diagram plot and label the following complex numbers: (4 marks)

(i) $z_1 = iz$

(ii) $z_2 = \overline{z}$

(iii) $z_3 = z^2$

(iv) $z_4 = z \cdot \overline{z}$

Question 2 **(6 marks)**

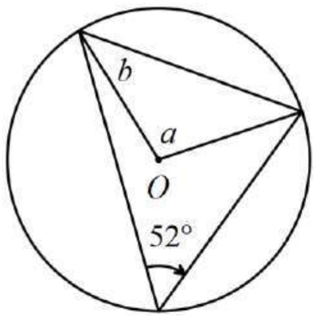
A boat is motoring at 18 km/h on a bearing of 245°. To an observer on board the boat, the wind appears to be blowing at 28 km/h from due west.

If the boat turned through 180° and increased its speed to 20 km/h, find the new apparent wind speed and direction.

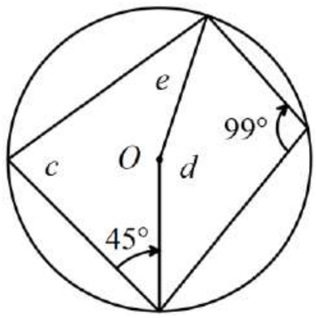
Question 3 **(8 marks)**

(a) In the following diagrams, O is the centre of the circle shown.

(i) Determine the values of a and b. (2 marks)

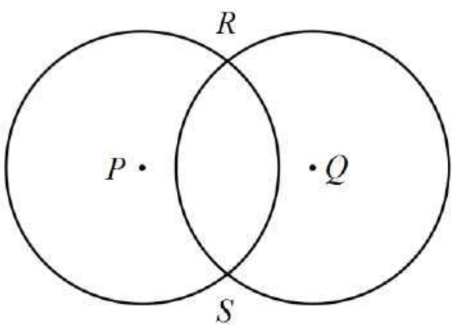


(ii) Determine the values of c, d and e. (3 marks)



(b) Two circles, both with radius 9 cm, and centres P and Q that are 12 cm apart, intersect at R and S as shown. Determine the exact area of the quadrilateral $PRQS$.

(3 marks)



Question 4 **(7 marks)**

(a) Let matrix $A = \begin{bmatrix} 2 & -2 \\ 7 & -6 \end{bmatrix}$.

(i) Determine A^{-1}. (1 mark)

(ii) Express the equations $7a - 6b = 23$ and $2a - 2b = 7$ as a system of matrices. (1 mark)

(iii) Show use of your answer from (i) to solve the matrix equation in (ii). (2 marks)

(b) Solve the equation $\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} B = B + \begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$ for the 2×2 matrix B. (3 marks)

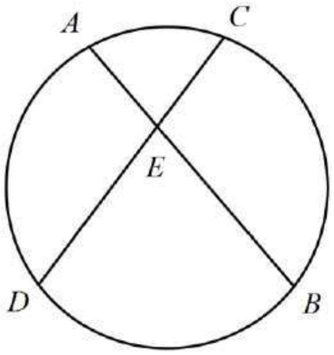
Question 5 **(7 marks)**

(a) In triangles ABC and DEF, $AC \cong DF$ and $\angle A \cong \angle D$. Is the additional fact that $BC \cong EF$ enough to prove that triangle ABC is congruent with triangle DEF ? Justify your answer.

(2 marks)

(b) In the circle shown below, not to scale, AB and CD are chords that intersect at E. If $AE = 4$ cm, $BE = 8$ cm and $CE = 6$ cm, determine the length of DE. Justify your answer.

(3 marks)



(c) Consider the true statement 'if a quadrilateral is a square then all four sides of the quadrilateral are the same length'. Write the converse of this statement and explain whether or not the converse is also true. (2 marks)

Question 6 **(8 marks)**

(a) Determine the sum of all the numbers contained in the row(s) of Pascals triangle in which the number 10 appears. (3 marks)

(b) Nine students applied for four temporary positions working for a cleaning company in their holidays.

(i) How many different selections of students could the cleaning company make to fill the four positions? (1 mark)

(ii) If six of the nine applicants were male, and the company wanted to employ an equal number of males and females, in how many ways could they do this? (2 marks)

(iii) Determine the number of ways that the application forms can be sorted into order if the six male applications must be kept together. (2 marks)

Question 7 **(7 marks)**

(a) A triangle with vertices at $A(1,\ 1)$, $B(3,\ 1)$ and $C(3,\ 4)$ is reflected in the x-axis and then rotated $90°$ anticlockwise about the origin.

(i) Find the matrix T that will combine these two transformations in the order given. (3 marks)

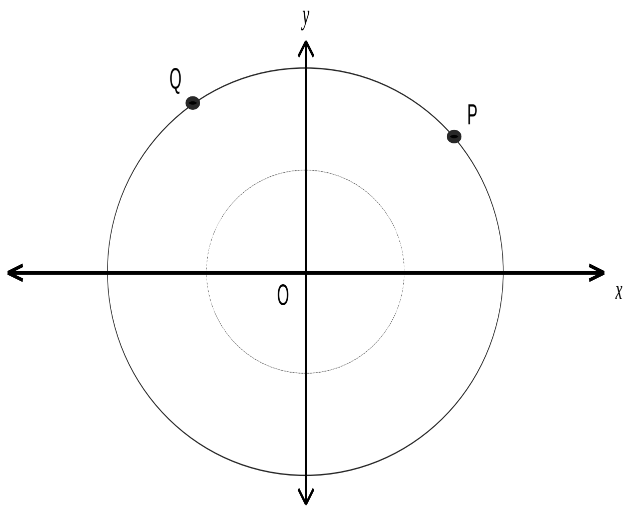
(ii) Find the coordinates of C after transformation by T. (1 mark)

(b) Another transformation matrix is given by $R = \begin{bmatrix} -0.6 & 0 \\ -1.2 & -0.6 \end{bmatrix}$.

Determine the area of triangle ABC after transformation by T and then by R. (3 marks)

Question 8 **(6 marks)**

The points P and Q lie on a circle of radius r and have polar coordinates (r,θ) and (r,ϕ) respectively, where $0<\theta<\phi<360°$.



(a) Express both of the vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in the form $a\mathbf{i}+b\mathbf{j}$. (2 marks)

(b) Use your answers from (a) to show that $\overrightarrow{OP}\bullet\overrightarrow{OQ}=r^2(\cos\theta\cdot\cos\phi+\sin\theta\cdot\sin\phi)$. (1 mark)

(c) Use the diagram above to state the size of $\angle POQ$. (1 mark)

(d) Use the definition $\mathbf{a}\bullet\mathbf{b}=|\mathbf{a}|\cdot|\mathbf{b}|\cos\theta$ and the result in (b) to show $\cos(\phi-\theta)=\cos\phi\cdot\cos\theta+\sin\phi\cdot\sin\theta$.

(2 marks)

Question 9 **(6 marks)**

In a new office building, the manager's office is to have one desk, three filing cabinets, one fax, one telephone and four chairs.
The supervisors are to have one desk, two filing cabinets, no fax, one telephone and two chairs.
The clerks are to have one desk, one filing cabinet, no fax, one telephone and one chair.

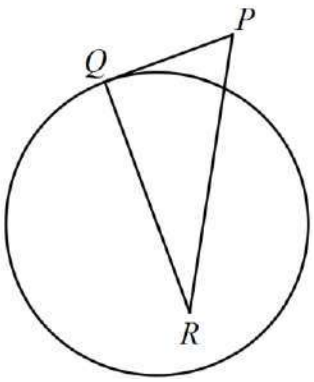
(a) Express this information in matrix M. (1 mark)

(b) In the new building there are to be 6 managers, 12 supervisors and 40 clerks. Express this information in matrix N and then show use of a matrix operation to determine the total number of desks, filing cabinets, fax machines, telephones and chairs needed. (3 marks)

(c) The cost of a desk, a filing cabinet, a fax, a telephone and a chair are \$250, \$190, \$125, \$85 and \$160 respectively. Express these costs in matrix C and then show use of a matrix operation to determine the total cost of all new furniture and office machines. (2 marks)

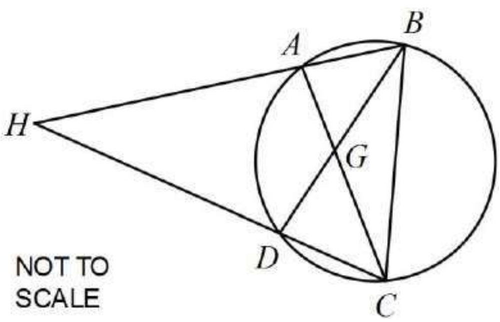
Question 10 **(9 marks)**

(a) The circle in the diagram has a diameter of 20 cm and PQ is a tangent to the circle at Q. If $PQ = 7.5$ cm, $QR = 18$ cm and $PR = 19.5$ cm, prove that R lies on the diameter of the circle. (3 marks)



(b) The points A, B, C and D lie on a circle of radius r. The lines AC and BD intersect at G. The lines BA and CD are produced to meet at H. $HAGD$ is a cyclic quadrilateral.

(i) Determine, with reasons, the size of $\angle BAC$. (4 marks)



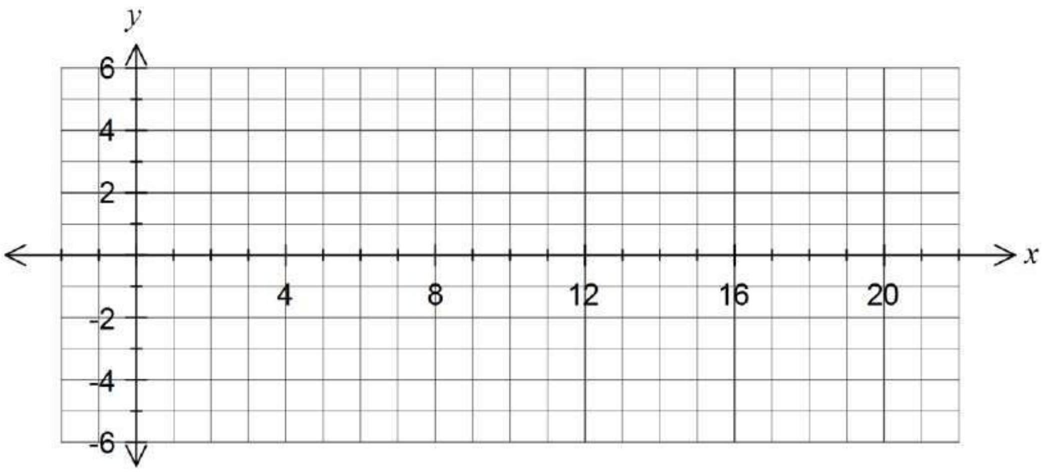
(ii) Determine, with reasons, the length BC in terms of r. (2 marks)

Question 11 **(9 marks)**

The motion of a small body moving along a straight track was recorded by a video camera for 20 seconds. An analysis of the motion showed that the distance, x cm, of the body from a fixed point O on its path t seconds after recording began was given by $x(t) = 3\cos\frac{\pi t}{4} - 4\sin\frac{\pi t}{4}$.

(a) The distance can also be given by $x(t) = a\sin\left(\frac{\pi t}{4} + b\right)$, where a and b are real constants. Determine the values of a and b. (2 marks)

(b) Graph $y = x(t)$ on the axes below for $0 \le t \le 20$. (3 marks)



(c) State the period and amplitude of the graph of $y = x(t)$. (2 marks)

(e) Determine the percentage of the first 20 seconds that the body was at least four cm away from the point O. (2 marks)

Question 12 **(12 marks)**

(a) Consider the expression $m^2 + 7$.

(i) Write down the values of $m^2 + 7$ for $m = 1, 3, 5, 7$ and 9. (1 mark)

(ii) Use your values from (a) to state the largest integer, p, that $m^2 + 7$ is always divisible by, when m is a positive odd integer. (1 mark)

(iii) Prove that $m^2 + 7$ is always divisible by p when m is a positive odd integer. (4 marks)

(b) Let $A = 2^{n+2} + 3^{2n+1}$, $n \in \mathbb{N}$.

(i) Show that $A = 7 \times 25325$ when $n = 5$. (1 mark)

(ii) Prove by induction that A is divisible by 7. (5 marks)

Question 13 **(8 marks)**

A helicopter, with a maximum speed through still air of 240 km/h, leaves its base at A to fly to a destination at B.

The position vector of B relative to A is $(155\mathbf{i} + 95\mathbf{j})$ km, and a steady wind of velocity $(-17\mathbf{i} - 22\mathbf{j})$ km/h is blowing over the area.

(a) Find the velocity vector the helicopter pilot should set in order to fly directly from A to B in the shortest time. (6 marks)

(b) What is the shortest journey time, to the nearest minute? (2 marks)