Example 7-12

A 1000-kg car travels at a constant speed of 100 km/h = 28 m/s = 62 mi/h up a 10% grade. (A 10% grade means that the road rises 1 m for each 10 m of horizontal distance—that is, the angle of inclination θ is given by $\tan\theta = 0.1$ [Figure 7-13].) What is the minimum power that must be delivered by the car's engine? (Neglect rolling friction and air drag.)

Figure 7-13

$\tan\theta \approx h/x \approx \sin\theta = h/s$

Picture the Problem The power delivered by the car's engine is the rate of decrease of its chemical energy. Some of it goes into increasing the potential energy of the car as it climbs the hill, and some goes into an increase in thermal energy, which is expelled as exhaust. For a 10% grade, $\tan\theta = 0.10$ is given, and $\sin\theta \approx \tan\theta$ because the angle is small (Figure 7-13). For the car–earth system, $W_{ext} = 0$, so the total energy is conserved.

1. The power input by the engine is the rate of decrease of its chemical energy:

$$P = -\frac{dE_{chem}}{dt}$$

2. The chemical energy change is found from the work–energy theorem:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm} + \Delta E_{chem} = 0$$

$$\Delta E_{chem} = -\Delta E_{mech} - \Delta E_{therm}$$

3. Convert the changes to time derivatives:

$$P = -\frac{dE_{chem}}{dt} = \frac{dE_{mech}}{dt} + \frac{dE_{therm}}{dt}$$

4. Since the speed $v = ds/dt$ is constant, the rate of change of the mechanical energy is just the rate of change of potential energy:

$$\frac{dE_{mech}}{dt} = \frac{dU}{dt} = \frac{d(mgh)}{dt} = mg\frac{dh}{dt}$$

5. From Figure 7-13 we can see that when the car travels a distance s along the road, it climbs a height h, which is related to s by:

$$h = s\sin\theta$$

6. We can use the approximation $\tan\theta \approx \sin\theta$ because the angle is small:

$$h = s\sin\theta \approx s\tan\theta = 0.1s$$

7. We can now relate the rate of change of mechanical energy to the speed:

$$\frac{dE_{mech}}{dt} = mg\frac{dh}{dt} = 0.1\,mg\frac{ds}{dt} = 0.1\,mgv$$

8. Substitute these results into the equation for power in step 3:

$$P = \frac{dE_{mech}}{dt} + \frac{dE_{therm}}{dt}$$

$$= 0.1mgv + \frac{dE_{therm}}{dt}$$

$$= (0.1)(1000\text{ kg})(9.81\text{ N/kg})(28\text{ m/s}) +$$

$$= 27.5\text{ kW} + \frac{dE_{therm}}{dt}$$

9. The minimum power occurs when $dE_{therm}/dt = 0$:

$$P_{min} = 27.5\text{ kW}$$