

Hmwk 13 (Supp)

③ Laplace in Spherical $V(r, \theta)$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) = \lambda(\lambda+1)$$

use method of Frobenius to show:

$$R(r) = A r^\lambda + B r^{-\lambda(\lambda+1)}$$

$$R(r) = \sum_{k=0}^{\infty} A_k r^{\lambda+k} \quad \text{Frobenius}$$

$$-\lambda(\lambda+1) + \sum_{k=0}^{\infty} A_k r^{\lambda+k} \frac{d}{dr} \left(r^2 \sum_{k=0}^{\infty} (\lambda+k) A_k r^{\lambda+k-1} \right) = 0$$

$$\sum_{k=0}^{\infty} (-\lambda(\lambda+1) A_k r^{\lambda+k}) + \frac{d}{dr} \left(\sum_{k=0}^{\infty} (\lambda+k) A_k r^{\lambda+k+1} \right) = 0 \quad (1)$$

$$\sum_{k=0}^{\infty} (-\lambda(\lambda+1) A_k r^{\lambda+k}) + \sum_{k=0}^{\infty} (\lambda+k)(\lambda+k+1) A_k r^{\lambda+k} = 0$$

$$\sum_{k=0}^{\infty} \left[-\lambda(\lambda+1) r^{\lambda+k} A_k + (\lambda+k)(\lambda+k+1) r^{\lambda+k} A_k \right] = 0$$

$$\sum_{k=0}^{\infty} \left[(-\lambda^2 - \lambda) r^{\lambda+k} A_k + (\lambda^2 + 2\lambda k + k^2 + k) r^{\lambda+k} A_k \right] = 0$$

$$\sum_{k=0}^{\infty} \left[A_k (x^2 + x + 2\lambda x - \lambda) r^{\lambda+k} \right] = 0$$

$$\Rightarrow A_0 (x^2 + x) r^x + A_1 (x^2 + 3x - 1) r^{1+x} + \dots = 0$$

coefficients must = 0

$$R(r) =$$