Test One Math 2310H, Fall 2004, Smith, Name:____________
(Use the backs of the pages. No calculators.)
I. a. Give the definition of what it means for a bounded function f on [a,b], to be "integrable" (in terms of upper and lower sums).

b. Give at least two different properties that guarantee a function on [a,b] is integrable. I.e. f is integrable on [a,b],

(i) if f is....

or (ii) if f is......

c. Is f(x) = e^x integrable on [0,1]? Why or why not?

d. Is f(x) = tan(x) integrable on [0,1]? Why or why not?

e. If f is integrable on [a,b] we know the integral of f can be expressed as a limit of "Riemann sums". Write out the expression for the integral of f(x) = x^3 on [a,b] as a limit of Riemann sums, explaining the meaning of the symbols you use, in terms of a subdivision: a = x(0) < x(1) < ...< x(i-1) < x(i) <....< x(n) = b.

II. Assuming the formula 1^3 + 2^3 +3^3 +…..+n^3 = n^4/4 + f4(n), where f4(n) is a polynomial in n of degree < 4, compute the integral of f(x) = x^3 on [a,b] as a limit of Riemann sums.

III. Define f(x) on [0,1] by setting f(x) = 1/3 for 0 ≤ x ≤ 1/2,
f(x) = 1/9 for 1/2 < x ≤ 3/4, f(x) = 1/27 for 3/4< x ≤ 7/8, etc...
so that f(x) = (1/3)^n for (2^(n-1) - 1)/2^(n-1) < x ≤ (2^n – 1)/2^n.

If we set f(1) = 0, then for a subdivision of [0,1] into 2n equal subintervals, the "right end point" Riemann sum equals (1/3)(1/2) + (1/9)(1/4) +.......+ (1/3^n)(1/2^n), where we have simplified the sum by combining the terms where f has the same value.

(i) Explain why f is integrable on [0,1].
(ii) Evaluate the integral of f on [0,1], explaining your method.

IV. We know that the integral of $f(x) = 1/x$ for $1 \leq x \leq 2$, equals $\ln(2)$. Use this fact to estimate $\ln(2)$. I.e. compute upper and lower sums for the integral, using a subdivision into two equal subintervals, to show that $(7/12) \leq \ln(2) \leq (5/6)$.

EXTRA: Ask and answer as interesting a question about the material of this course as possible.