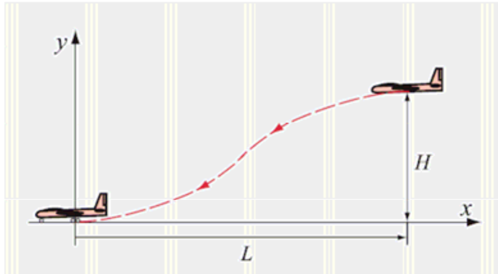
**2.26** An aircraft begins its descent at a distance $x = L$ ($x = 0$ is the spot at which the plane touches down) and an altitude of $H$. Suppose a cubic polynomial of the following form is used to describe the landing:

$$y = ax^3 + bx^2 + cx + d$$

where $y$ is the altitude and x is the horizontal distance to the aircraft. The aircraft begins its descent from a level position, and lands at a level position.




($a$) Solve for the coefficients $a$, $b$, $c$, and $d$.

($b$) If the aircraft maintains a constant forward speed ($\frac{dx}{dt} = u = constant$) and the magnitude of the vertical acceleration ($\frac{d^2y}{dt^2}$) is not to exceed a constant $A$, show that $\frac{6Hu^2}{L^2} \le A$.

($c$) If $A = 0.3\ \text{ft/s}^2$, $H = 10000$ ft, and $u = 150$ mph, how far from the airport should the pilot begin the descent?

**Solution**

($a$) Since the origin is at the point of landing, $y = 0$ when $x = 0$. This means that $d = 0$. Since the aircraft begins its descent from a level position and lands at a level position, $\left.\frac{dy}{dx}\right|_{x=0} = 0$ and $\left.\frac{dy}{dx}\right|_{x=L} = 0$. This means that $c = 0$ and $3aL^2 + 2bL = 0$, or $b = -\frac{3}{2}aL$. With these determinations, the polynomial that describes the landing becomes:

$$y = ax^3 - \frac{3aL}{2}x^2$$

Finally, $y = H$ when $x = L$, which yields: $H = aL^3 - \frac{3aL}{2}L^2 = -\frac{aL^3}{2}$. Solving for $a$, gives: $a = -\frac{2H}{L^3}$. Substituting into the polynomial, we have the final form:

$$y = \frac{3H}{L^2}x^2 - \frac{2H}{L^3}x^3$$

($b$) Differentiating the result from part ($a$) twice, we have:

$$\frac{dy}{dt} = \frac{6Hx}{L^2}\frac{dx}{dt} - \frac{6Hx^2}{L^3}\frac{dx}{dt}$$

But, $\frac{dx}{dt} = u = constant$ so that $\frac{dy}{dt} = \frac{6Hxu}{L^2} - \frac{6Hx^2u}{L^3}$. Differentiating again, $\frac{d^2y}{dt^2} = \frac{6Hu^2}{L^2} - \frac{12Hxu^2}{L^3}$. Now, $\frac{d^2y}{dt^2} \le A$ so that $\frac{d^2y}{dt^2} = \frac{6Hu^2}{L^2} - \frac{12Hxu^2}{L^3} \le A$. Since $x > 0$, this implies that $\frac{6Hu^2}{L^2} \le A$.

($c$) From part ($b$), $L \ge \sqrt{\frac{6Hu^2}{A}}$. Substituting $A = 0.3\ \text{ft/s}^2$, $H = 10000$ ft, and $u = 150$ mph, yields:

$$L \ge \sqrt{\frac{6(10000\ \text{ft})(150\ \text{mph})^2}{(0.3\ \text{ft/s}^2)} \cdot \left(\frac{5280\ \text{ft}}{\text{mile}} \cdot \frac{\text{hr}}{3600\ \text{s}}\right)^2} = 98387\ \text{ft} = 18.6\ \text{miles}$$