where $\boldsymbol{\beta}'_1$ and $\boldsymbol{\beta}'_2$ are the velocities of m_1 and m_2, respectively, in the C-O-M frame.

The boost, $\boldsymbol{\beta}'$, needed to go from the laboratory to the C-O-M frame, has the value

$$\boldsymbol{\beta}'_2 = -\boldsymbol{\beta}'. \tag{7.81$'$}$$

Since all velocities are parallel, the velocity addition formula Eq. (7.15) gives the velocity $\boldsymbol{\beta}'_1$ of mass m_1 in the C-O-M system in terms of $\boldsymbol{\beta}'$ and its velocity $\boldsymbol{\beta} = \mathbf{v}/c$ in the laboratory frame,

$$\beta'_1 = \frac{\beta - \beta'}{1 - \beta\beta'}. \tag{7.82}$$

The total squared momentum in the C-O-M frame given in Eq. (7.80) can be rewritten using the results of Eqs. (7.81) and (7.82) as

$$p^\mu p_\mu = \frac{m_2^2\beta^2(1-\beta'^2)c^2}{\beta - \beta'}. \tag{7.83}$$

Equating Eqs. (7.79$'$) and (7.83) gives a single equation that can be solved for the boost velocity β'. There are two real roots, one of which corresponds to the physically meaningful case of $\beta' < 1$.

Since the spatial momentum in the C-O-M frame is zero, there is clearly more energy, p^0, in this frame than in the laboratory frame.* The excess energy in the C-O-M frame, ΔE, is obtained by subtracting the time component of Eq. (7.79) from the time component of Eq. (7.80).

The total momentum four vector is conserved, which automatically implies both conservation of spatial linear momentum and conservation of total energy (including rest mass energy). Our major tools for making use of the conservation principle are Lorentz transformations to and from the C-O-M system, and the formation of Lorentz invariants (world scalars) having the same value in all Lorentz frames. Since energy and momentum are combined into one conservation law, the relativistic results are more easily obtained than the nonrelativistic results of previous chapters. The transformations between laboratory system and C-O-M system are merely special cases of the Lorentz transformation.

As an example of the use of Lorentz invariants, let us consider a reaction initiated by two particles that produces another set of particles with masses m_r, $r = 3, 4, 5, \ldots$. In the C-O-M system, the transformed total momentum is

$$p^{\mu'} = (E'/c, 0, 0, 0). \tag{7.84}$$

It is often convenient to look on the C-O-M system as the proper (or rest) system of a composite mass particle of mass $M = E'/c^2$.† The square of the magnitude of

*For a single particle, the energy has a minimum value, mc^2, in the rest frame. The C-O-M frame is not the rest frame of either particle.

†Although it is customary in high-energy physics to use units in which $c = 1$, it seems more helpful in an introductory exposition such as this to retain the powers of c throughout.