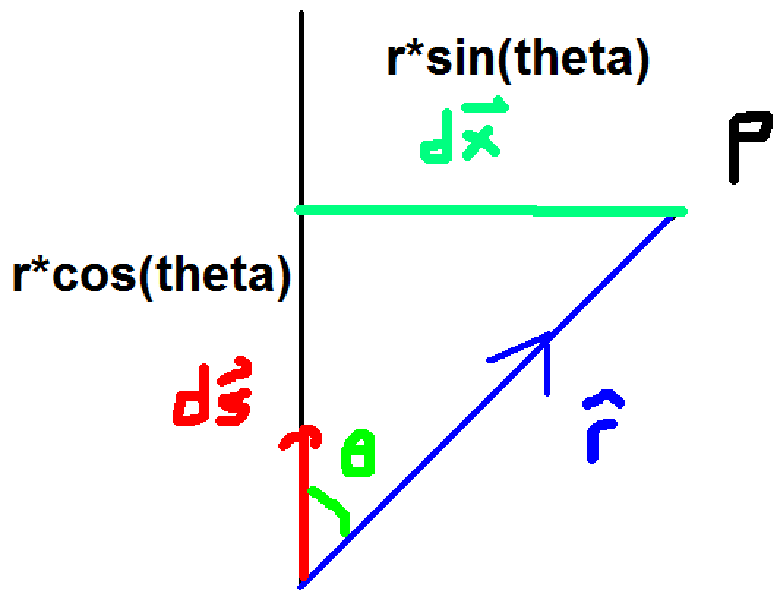

Figure 1: Problem diagram

Start with Eq. (1), the Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2} \tag{1}$$

From the diagram:

$$r = \sqrt{x^2+y^2}$$
$$x = r\sin(\theta)$$
$$y = r\cos(\theta)$$
$$d\vec{s} = d\vec{y}$$

Substitute into Eq. (1):

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{y}\times\hat{r}}{x^2+y^2} \tag{2}$$

Taking the cross product, $id\vec{y}\times\hat{r} = dy\sin(\theta)(-\hat{k})$. Substitute in:

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{dy\sin(\theta)}{x^2+y^2}(-\hat{k}) \tag{3}$$

From above, $\sin(\theta) = \frac{x}{r}$:

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{xdy}{(x^2+y^2)^{3/2}}(-\hat{k}) \tag{4}$$

Find $\vec{B}$ by taking the integral over the infinite line:

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{xdy}{(x^2+y^2)^{3/2}}(-\hat{k}) \tag{5}$$

$$\vec{B} = 2\frac{\mu_0 i}{4\pi}\int_0^\infty \frac{xdy}{(x^2+y^2)^{3/2}}(-\hat{k}) \tag{6}$$

Integrate (I used Wolfram|Alpha) to get:

$$\vec{B} = 2\frac{\mu_0 i}{4\pi}\left[\frac{y}{x\sqrt{x^2+y^2}}\right]_0^\infty(-\hat{k}) \tag{7}$$

The indefinite integral evaluates to:

$$\vec{B} = \frac{\mu_0 i}{2\pi x}(-\hat{k}) \tag{8}$$

We can confirm this using Ampere's Law (left as an exercise to the reader...)