To do calculations, just do a Google search for a cosmological calculator. Here is one by Ned Wright:

http://www.astro.ucla.edu/~wright/CosmoCalc.html

Bold writing is information that I've added

Terms:

CMB rest: There is an FAQ entry for this. An observer at rest relative to the CMB sees approximately the same temperature (of the ancient light) in all directions. There is no Doppler hotspot which would indicate that he or she was moving in that direction. It's like being at rest with respect to the ancient matter when it was more uniformly spread out, or with respect to the expansion process itself.

Refereced FAQ entry:

The Cosmic Microwave Background is remarkably uniform -- the temperature of the light is the same from all directions in the sky to within about one thousandth of one percent!

That is, if you first adjust for the effects of solar system/orbital motion.

Solar system motion (including motion within the solar system of whatever instrument is mapping temperature) causes a Doppler hot-spot to appear in the direction of motion, warmer by over a hundredth of one percent -- ten times more than the variation seen otherwise. There is a cold-spot in the opposite direction. The temperature data is adjusted to get rid of the effect of this "Doppler dipole" effect of the instrument's own motion relative to the ancient light. Our maps of the CMB temperature represent the microwave sky as it would appear to an observer at rest relative to the CMB, for whom, in other words, there is no Doppler dipole due to his or her individual motion.

The idea of an observer being at CMB rest is identical to the idea of being at rest relative to the "Hubble flow," i.e., relative to the average motion of nearby galaxies. All galaxies are approximately at CMB rest, so that galaxies A and B can both be "at rest" by the CMB definition, and yet the distance between them is increasing. One way of verbally describing this is that the space between the galaxies is expanding.

The concept of CMB rest comes in handy in many ways. For example, when we say that the universe is 13.8 billion years old, we're referring to the time measured on a hypothetical clock that is in a state of CMB rest.

Universe time: Time as clocked by observers at CMB rest.

Proper distance at a particular time t: What you would measure by any conventional means (radar, tape measure...) if you could stop expansion at some given moment of universe time. Stopping expansion gives you time to measure---the distance won't change while you are sending the radar pulse, for example.

Scale factor a(t): This curve plots the expansion of distance as an increasing function of time. It is normalized to equal 1 at the present time. a(now) = 1. Back when distances between stationary

observers were only half what they are today a(then) = 0.5. The slope a'(t) has not been constant so it's convenient to have the curve as a record of expansion history. Picture:
http://ned.ipac.caltech.edu/level5/M...s/figure14.jpg
The dark solid curve labeled (.27, .73) is the one to focus on.

One way to represent the scale factor is as a(t) = (d(t))/(d_0), where d(t) is the distance in between ANY to arbitrary points in intergalactic space (which could be two galaxies, if you like.) at some time t, and d_0 is the distance between the same two points at some earlier time. So, the scale factor represents how much larger the distance between our two points is now (d(t)) compared to some time in the past (d_0). This this is increasing with time, we can understand the expansion - the distance in between any two arbitrary points in the Hubble Flow (expanding space, the space in between galaxies) increases over time.

Fractional rate of increase of a(t): A good handle on the rate distances are increasing is the fractional or percentage increase over time. Currently the scalefactor increases by about 1/140 of one percent per million years. So any largescale distance (e.g. between galaxies free of each other's gravity and each approximately at CMB rest,) will increase at that rate. (More precisely using the latest data 1/139 of one percent per million years.) The math expression for this rate, at any time t, is a'(t)/a(t). This is the absolute increase at that time, divided by the current size at that time, IOW a fractional or percentage increase rate.

Hubble rate H(t): By definition H(t) = a'(t)/a(t), just another name for the fractional rate of expansion. The current value of the Hubble rate is denoted Ho. Or you could say H(now), or a'(now)/a(now). It would all mean the same thing. Mathematically it is a fractional rate of increase the current value of which is 1/140 of one percent per million years. (Or 1/139 using the latest data)
That's the rate that distances (between observers at CMB rest) grow, at present. Using proper distance and the universe standard timescale.
In common astronomy units it is 70.4 km/s per Mpc. 70.4 km/s is the speed a distance of one Mpc is growing.
The Hubble rate is slated to decline in future to sqrt 0.728*Ho ≈ 60 km/s per Mpc.

One important concept in cosmology is apparent recessional velocity. As is being explained, the reason galaxies all appear to be moving away from is because the universe is expanding, not because the galaxies are actually moving. However, it is sometimes useful to model the universe in a way in which your galaxy is at rest, at the 'center' of the universe, and all other galaxies are moving away (local coordinates). Of course, there isn't any importance to this, it's just convenient.

So, this apparent recessional velocity can be calculated using the above Hubble rate, via Hubble's Law, V = H(t)D, where D is the distance to the galaxy. This makes sense - farther galaxies appear to be moving faster. That's because their light is more redshifted, which is due to the gfact photons from those galaxies must cross MORE expanding space, causing their wavelengths to be increased, and hence be more redshifted.

Hubble radius c/H(t): This is the radius within which proper distances increase at speeds less than c. If a photon is trying to get to us and can manage to get within this radius then it will begin to approach. The photon's own speed is then faster than the remaining distance is increasing, so it can make progress towards us and narrow the gap.
The google search window doubles as a calculator. Try using it to find the current Hubble radius in

lightyears. I invite you to copy this into the search window:
1/70.4 km/s per Mpc
When you press return, the calculator will say 13.9 billion years.
Multiply by c and you obviously get 13.9 billion lightyears.
This is the current Hubble radius.
Photons within that radius are going to make it.

Cosmic Event Horizon ≈ c/(sqrt 0.728*Ho) ≈ 16 billion lightyears.
Photons heading for us can still make it even if they are OUTSIDE the current Hubble radius as long as the radius itself is increasing fast enough and reaches out and takes them in.
What would make c/H(t) increase? The denominator H(t) decreasing would. The Hubble expansion rate has decreased sharply in the past which is why we can see such a lot of stuff that we know is receding faster than light.
But according to the standard cosmic model H(t) though still declining is not expected to go below sqrt(0.728) of its current value.
It is expected to level out at (sqrt 0.728)*70.4 km/s per Mpc
So what will the Hubble radius be then?
Try putting this in the google window
c/(sqrt 0.728 *70.4 km/s per Mpc) in lightyears
You will get the longterm value of the Cosmic Event Horizon (abbreviated CEH)

*The number 0.728 is technical and hard to explain, so I've had to put it in *ad hoc*. It represents a constant VACUUM CURVATURE contributing to the near flatness of space, which would otherwise be negatively curved (e.g. triangles adding to less than 180 degrees). Without such an inherent constant curvature bias, (or cosmological constant) the current density of matter/energy would only be 0.272 (or about 27%) of what was needed for the observed degree of flatness. So (although in my opinion it's a bit confusing to think this way) the number 0.728 could be imagined as a fictitious energy contribution making up the rest of what would be needed without a cosmological constant.
The square root of 0.728 gets into the picture for technical reasons when we want to talk about the longterm value of the Hubble rate, the level below which it is not expected to decline (because of the acceleration in the scalefactor.)

Curvature of the universe - if you extended to lines an extremely far distance, would they stay parallel, converge, or diverge? If they stay parallel, we say the universe is FLAT, or that it has zero spatial curvature. If they converge (like on the surface of a sphere, think of lines of meridian converging at the North Pole), then the universe is said to be CLOSED, or to have positive spatial curvature. For a lower dimensional ANALOGY, a positively curved surface would be a sphere, like the surface of the earth. However, this is just an analogy. If the lines diverge, the universe is said to HYPERBOLIC, or to have negative spatial curvature. A lower dimensional analogy would be a saddle.

The Cosmological Constant, or the greek letter LAMBDA, refers to whatever is causing the expansion of the universe - general relativity allows you to do this in the form of a constant that gets plugged into the Einstein Field equation. However, note that the cosmological constant is physically equivalent to a vacuum energy that has NEGATIVE pressure - the latter is how it is usually represented, and also note that the pressure is the negative of the energy density, which remains constant, so that the strength remains constant, and it can continue to accelerate the expansion of that universe. However, it may not be vacuum energy, and just a geometrical feature of the universe. It is key to remember that for calculational purposes, they are

equivalent.

Marcus's take on the CC:

Lambda is an inherent minimal growth rate that nature's geometry has a built-in tendency towards. For most of history because it got such a terrific kickoff at the start, the growth rate has been much bigger. but now it is settling down. As the density thins out, H^2 is getting closer and closer to H_∞^2
That's what the Friedmann equation says. The amount it has left to go is proportional to the density
$H^2 - H_\infty^2 = (8\pi G/3)\rho$
and as distances (and volumes) enlarge, the density gets less and less.

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The geometric relations among things are not a physical substance. "Space" is a word which does not refer to a material. It refers to a bunch of geometric relationships.

What marcus is trying to convey here is that it is misleading to say that space is 'stretching'. Really, the *metric* (the way we define distance) is changing - the distance in between points increases, but that's all we have to say - the points themselves do not have to move, and the interveneing space doesn't really 'expand'. It's just that the geometry changes. If the two points were seperated by a ruler, you would find at a later time that the ruler didn't measure the full distance.

==

It has been calculated what the maximum distance some photons, a flash of light, could now be from the sender, if the flash is sent at start of expansion or as close to then as you like.
So the flash has been traveling for the whole 13.7 billion year history of expansion. (Today some cosmo models go back before start of expansion into a contraction phase, but we arent including that, just the usual 13.7 billion year expansion age.)

That maximum distance is called the PARTICLE HORIZON and it is calculated to be about 46 billion lightyears. The farthest a flash of light can have gotten (with the help of expansion) in the whole 13.7 billion year history is only 46 billion lightyears. We're fairly sure now that the circumference of the entire U, if it isn't actually infinite, is considerably bigger than that, by over a factor of 10.

This just means that a flash of light (which defines the fastest method of information transfer) emitted at the beginning of the expansion of the universe could only travel so far (since it has a finite speed). Normally, since the universe is 13.7 billion years old, you'd conclude that this 13.7 billion light years. However, this is where expansion comes in - the light must travel a much farther distance. If you were running across a field, and the field was growing, you will have ended up traveling much farther than if the field wasn't expanding. That's why particle horizon is 46 billion light years away, not 13.7.

==

Regarding Loop Quantum Cosmology and a bit about inflation:

One type model that is gaining attention involves a BOUNCE. In the main model of this type you get a brief period of faster than exponential growth of distances. Normally what is called "inflation" by cosmologists is exponential and slightly slower growth.

a(t) = eHt with H either steady or slowly declining.

In stark contrast to this, in socalled Loop cosmology you get this but with H increasing very rapidly to extremely high (Planck scale) values and because this is faster than the unsual exponential growth called inflation it is called "superinflation" by Loop cosmolgy researchers. I don't remember hearing the term "hyperinflation" in cosmology.

For me it is completely speculative what conditions could have been like and what could have been happening at such extremely high densities. In Loop cosmology, according to their equation model, gravity becomes repellent at near Planck density, which is what causes the bounce. It is a quantum gravity effect. Quantum nature doesn't like to be pinned down too tightly, so resists extremely dense compression.

One option is not to try to understand the very beginning of expansion but only start thinking about it a few blinks after it started. Wait until there is more agreement among the real experts before trying to understand. sorry so unhelpful.

==

More information about specification calculations:

You are probably right, esp about respecting conventional notation. I like the idea that the Hubble radius (that threshhold of admission for photons trying to get to us) is so important and one can just flip the Hubble constant and get it

H0 = 1/13.9 ppb per year ≈ 0.072 ppb per year
Hubble radius (now) = 13.9 billion lightyears.

You get to remember two quantities for the price of remembering one. But it does jar a little to write 1/13.9.

I will calculate the limiting value of the Cosmic Event Horizon and of the Hubble radius and try writing it in the style you suggest. Let's use the current estimates of 70.4 km/s per Mpc and 0.728

Put this into the google search window:
1/(sqrt(0.728)*70.4 km/s per Mpc)
16.279 billion years

So, without doing much round-off, what we get for for the longterm Hubble radius is
16.279 billion lightyears.

And what we get for the longterm Hubble rate is 1/16.279 or written as you suggest:
H∞ = 0.0614 ppb per year.

As I say, you are probably right. But for a while, at least, to see how it goes, I will keep trying to think of it as
Radius = 16 billion lightyears
H∞ = 1/16 ppb per year,

also let's keep the option of adding another digit of accuracy---e.g. say 16.3 and 1/16.3

===

There is a form of the Friedmann equation (for spatially flat or nearly flat universe) which goes like this:

$H^2 - H_\infty^2 = (8\pi G/3)\rho m$

At any given time H(t) is going to be bigger than its eventual value in such amount that their SQUARES differ by something proportional to the current matter density (radiation, dark and ordinary matter combined).

The constant (8πG/3) we can't do anything about, it stems from the original Einstein GR equation. Basically the equation says the denser the matter load, the more rapid expansion must be to maintain balance and keep things on the level. The more thinly matter is spread, on the other hand, the closer H can come down to its eventual longterm value.

If you solve that for the critical mass density that just balances the current expansion rate, the lefthand side comes out
.272*(70.4 km/s per Mpc)^2 (Put that in google window)
Then if you divide both sides by (8pi*G) you get
.272*(70.4 km/s per Mpc)^2/(8pi*G/3)
But that is expressed in kilograms per cubic meter and it is such a tiny mass density that it is hard to remember, so let's use the ENERGY DENSITY EQUIVALENT of the mass density and multiply by c2.
Then the critical matter density comes out
.272*(70.4 km/s per Mpc)^2/(8pi*G/3)*c^2 (Paste that into the window.)

What you get is 0.2276 nanojoules per cubic meter, or as the google calculator likes to say it: 0.2276 nanopascals. Used as a measure of energy density, one pascal = one joule per m3

So as not to overstate the precision, we could say 0.23 nanopascal for the critical matter density.

BOLD:More about this:

You asked about where the calculated density comes from. I derived it in post #301.
The density is normally denoted by the Greek letter rho (ρ) and it is of ordinary matter+dark matter+radiant energy all combined in one. At present the contribution from radiation is negligible so we call it simply "matter density".

The basic equation of cosmology is called the Friedmann equation, and it is derived from Einstein GR equation after making some simplifying assumptions. Cosmo is a mathematical science so it is all about equations, not about VERBAL explanations. So to understand where the density comes from we have to look at the Friedmann equation.

Assuming spatial flatness, or near-flatness, this takes a rather simple form:

$H^2 - H_\infty^2 = (8\pi G/3)\rho$

Here rho is the density I was talking about. And H is a number-per-unit-time which is today's fractional growth rate of distance. And H∞ is another number-per-unit-time which is the fractional growth rate of distance in the far future, which the universe is heading towards. Its square is the same as the cosmo constant Lambda except for a factor of c2/3. So we can treat it as a look-alike or stand-in for λ.

Now H and H∞ are things that cosmologists infer from measurement. Both are small fractions of a percent growth per million years. They are, respectively, estimated to be 1/139 and 1/163 of one percent per million years.

So you could say that the critical density ρ needed for perfect flatness is based on both H and H_∞.

But you could also say that all THREE quantities are based on the millions of datapoints of observation that has accumulated. Because estimates of all three are adjusted to FIT THE DATA. In a mathematical science you adjust the parameters of the model to fit the data. The Friedmann equation model is a simple version of Einstein GR which has been checked in many different situations (solar system, neutron stars, precision satellites with clocks or gyroscopes, galaxy counts, microwave background etc.)

So for the time being we trust the Friedmann equation and we adjust all the parameters together to get the best fit.

Note: Wherever you see an H∞ H_∞ is implied. The subscript is lost when copying and pasting from the forum post. Same goes for H2 and H∞2, they both mean H^2 and (H_∞)^2 are implied.

Even more:

Let's take another look at our basic expansion-rate equation
$H2 - H\infty 2 = (8\pi G/3)\rho$

Just using Freshman calculus we can differentiate it, and some nice things happen. The constant term drops out and we just have.

$2HH' = (8\pi G/3)\rho'$

But density ρ is essentially just some mass M divided by an expanding volume proportional to the cube of the scalefactor: a^3
$(M/a3)' = -3(M/a4)a' = -3\rho(a'/a) = -3\rho H$
Because by definition $H = a'/a$

$2HH' = (8\pi G/3)(-3\rho H) = - 8\pi G\rho H$, and we can cancel 2H to get:

$H' = - 4\pi G\rho$

I've highlighted that because it comes in a few lines later. Again by definition $H = a'/a$ so we can approach H' from another direction:
$H' = (a'/a)' = a''/a - (a'/a)2 = a''/a - H2$
It's great how much of the first 2 or 3 weeks of a beginning calculus course comes into play: chain rule, product rule, (1/xn)'...

Now the Friedman equation tells us we can replace H2 by $H\infty 2 + (8\pi G/3)\rho$. So we have
$H' = a''/a - H2 = a''/a - H\infty 2 - (8\pi G/3)\rho = - 4\pi G\rho$

Now we group geometry on the left and matter on the right, as usual, and get:
$a''/a - H\infty 2 = (8\pi G/3)\rho - 4\pi G\rho = - (4\pi G/3)\rho$
Here we used the arithmetic that $8/3 - 4 = - 4/3$

This is the socalled "second Friedmann equation" in the matter-dominated case where pressure is neglected.
$a''/a - H\infty 2 = - (4\pi G/3)\rho$

We can make another application of our basic Friedmann equation to replace
$(4\pi G/3)\rho$ by (H^2 - H_∞^2)/2
$a''/a = H\infty 2 - (4\pi G/3)\rho$ = H_∞^2 - (H^2 - H_∞^2)/2
= (3H_∞2 - H^2)/2

This will tell us the time in history when the INFLECTION occurred. When the distance growth curve slope stopped declining and began to increase. This is the moment when a" = 0. It marks when actual acceleration of distance growth began---i.e. when a" became positive.
To find that time all we need to do is find when
H^2 = 3H_∞^2
since then their difference will be zero, making a" = 0. That means
$H = \sqrt{3}$ H_∞ = $\sqrt{3}$/163 percent per million years = 1/94 percent per million years.
That happened a little less than 7 billion years ago. In other words when expansion was a bit less than 7 billion years old. You can see that from the table. The 1/94 fits right in between 6 billion years ago and 7 billion years ago. In between 1/100 and 1/90.

Code:
Standard model -- WMAP parameters (distances in Gly)

time(Gyr)	z	H(conv)	H(d-1)	Hub-radius	dist-now	dist-then
0	0.000	70.4	1/139	13.9	0.0	0.0
1	0.076	72.7	1/134	13.4	1.04	0.97
2	0.161	75.6	1/129	12.9	2.16	1.86
3	0.256	79.2	1/123	12.3	3.36	2.68
4	0.365	83.9	1/117	11.7	4.67	3.42
5	0.492	89.9	1/109	10.9	6.10	4.09
6	0.642	97.9	1/100	10.0	7.66	4.67
7	0.824	108.6	1/90	9.0	9.39	5.15
8	1.054	123.7	1/79	7.9	11.33	5.52
9	1.355	145.7	1/67	6.7	13.53	5.74
10	1.778	180.4	1/54	5.4	16.08	5.79
11	2.436	241.5	1/40	4.0	19.16	5.58
12	3.659	374.3	1/26	2.6	23.13	4.97
13	7.190	863.7	1/11	1.1	29.15	3.56
13.6	22.22	4122.8	1/2.37	0.237	36.69	1.58

The present Hubble rate is put at 70.4 km/s per Mpc which means distances between stationary observers increase 1/139 percent per million years. And the Hubble radius (a kind of threshhold within which distances are expanding slower than c) is currently 13.9 billion LY.

So by analogy you can see how the Hubble rate has been declining, while the Hubble radius (reciprocally) has extended out farther and farther.

More calculations using the Friedman equations:

By definition $H = a'/a$, the fractional rate of increase of the scalefactor.

We'll use ρ to stand for the combined mass density of dark matter, ordinary matter and radiation. In the early universe radiation played a dominant role but for most of expansion history the density has been matter-dominated with radiation making only a very small contribution to the total. Because of this, ρ goes as the reciprocal of volume. It's equal to some constant M divided by the cube of the scalefactor: M/a^3.

Differentiating, the constant term drops out.
$2HH' = (8\pi G/3)\rho'$
Then we use our formula for the density change:
$2HH' = (8\pi G/3)(-3\rho H) = - 8\pi G\rho H$, and we can cancel 2H to get the change in H, namely H':

$H' = - 4\pi G\rho$

I've highlighted that because it gets used a few lines later. Again by definition $H = a'/a$ so we can differentiate that by the quotient rule and find the change in H by another route:
$H' = (a'/a)' = a''/a - (a'/a)2 = a''/a - H2$

Now the Friedman equation tells us we can replace H2 by $H\infty2 + (8\pi G/3)\rho$. So we have
$H' = a''/a - H2 = a''/a - H\infty2 - (8\pi G/3)\rho = - 4\pi G\rho$

We group geometry on the left and matter on the right, as usual, and get:
$a''/a - H\infty2 = (8\pi G/3)\rho - 4\pi G\rho = - (4\pi G/3)\rho$
Here we used the arithmetic that $8/3 - 4 = - 4/3$

This is the so called "second Friedmann equation" in the matter-dominated case where radiation pressure is neglected.
$a''/a - H\infty2 = - (4\pi G/3)\rho$
In the early universe where light contributes largely to the overall density a radiation pressure term would be included and, instead of just ρ in the second Friedmann equation, we would have ρ+3p.

Now using the second Friedmann equation we would like to discover the time in history when the INFLECTION occurred. When the distance growth curve slope stopped declining and began to increase. This is the moment when a'' = 0. It marks when actual acceleration of distance growth began---i.e. when a'' became positive.

We can use the MAIN Friedmann equation to replace
$(4\pi G/3)\rho$ by $(H2 - H\infty2)/2$ in the second equation.
$a''/a = H\infty2 - (4\pi G/3)\rho = H\infty2 - (H2 - H\infty2)/2$
$= (3H\infty2 - H2)/2$

Now to find the inflection time, all we need to do is find when it was that
H2 = 3H∞2
since then their difference will be zero, making a" = 0. That means H = √3 H∞ = √3/163 percent per million years = 1/94 percent per million years. As one sees from the table, that happened a little less than 7 billion years ago. In other words when expansion was a bit less than 7 billion years old. You can see that from the table. The 1/94 fits right in between 6 billion years ago and 7 billion years ago. In between 1/100 and 1/90.

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In another thread someone had a question concerning the PRESENT rate of distance expansion in real terms--namely how much was it currently accelerating. So I wanted to show an easy way to address the question of acceleration and come up with a definite number.
Of course to get a definite speed one has to specify some particular distance between two stationary observers and see how fast that is growing and how much the growth is speeding up.

What distance one chooses to look at is somewhat arbitrary---I picked 13.9 billion lightyears because it makes the numbers simple. It is a bit less than a third of the current radius of the observable region.

For definiteness I use the key model parameters from the 2010 WMAP7 report by Komatsu et al, namely 0.272, 0.728, and 70.4 km/s per Mpc.
The current Hubble growth rate of 70.4 km/s per Mpc means distances between stationary observers are currently increasing by H = 1/139 of a percent per million years. And according to the standard model the limit that H is tending to is H∞ = 1/163 of a percent per million years*

The Friedmann equation model in the spatially flat case is
H2 - H∞2 = (8πG/3)ρ
where ρ is the density of all kinds of matter and radiation (excluding the cosmological constant, which I'm taking to be simply that: the cosmological constant.)
In the case where the contribution of radiation to ρ is small compared with that of dark and ordinary matter, the acceleration equation takes this form:
a"/a - H∞2 = - (4πG/3)ρ

So then we have:
a"/a = H∞2 - (4πG/3)ρ
and using the main Friedmann equation to replace (4πG/3)ρ by (H2 - H∞2)/2, we have:
a"/a = H∞2 - (H2 - H∞2)/2
a"/a = (3H∞2 - H2)/2, and factoring out H2 we get:
a"/a = [(3(H∞/H)2 - 1)/2]H2
a"/a = [(3(139/163)2 - 1)/2]H2
a"/a = 0.59 H2

Since we are asking about acceleration at the present time and by convention the scalefactor a(now) = 1 we can just write a" = 0.59 H2
and if we choose, as mentioned earlier, the distance R = 13.9 billion lightyears to be the present separation between the pair of stationary observers or objects then the acceleration is just gotten by multiplying on both sides by R:
a"R = 0.59 H2R

Now HR = c, because that's how R was chosen, and so
a"R = 0.59 Hc
This means that the current acceleration is 0.59/139 = 1/236 of a percent of the speed of light per million years.

I like this example because it gives an idea of how slow the acceleration is. The distance itself is currently increasing at the speed of light. And that rate is scarcely changing at all! Indeed after a million years it will still only be just slightly (a small fraction of a percent) larger than the speed of light.

*The relation to the cosmological constant is $H_\infty^2 = \Lambda c^2/3$