Tutorial on Basic Motion of an Object

Tools Required

- **A bit of calculus (e.g. integration and differentiation)**
- **A little algebra (e.g. solving quadratic equations, 2 equations in 2 unknowns)**
- **Some geometry and trigonometry (e.g. angles, sine, cosine, tangent, etc.)**
- **An open mind willing to learn**

Prologue

- **Physics of motion can be very confusing if you have equations provided to you and are not sure which ones apply nor how to use them to solve the problem at hand.**
- **Getting the 'right' answer is important, but understanding how to solve the problem (i.e. how you get the right answer) is just as important, if not more so.**
- **Most if not all elementary motion problems (e.g. free falling body, projectiles, incline planes) can be solved by properly following this simple methodology:**

  1) **Draw a free body diagram (FBD) depicting the object of interest and the forces acting on it. Include relevant items such as initial velocity or acceleration and necessary geometry (e.g. angle of an incline).**
  2) **Choose a convenient coordinate system. A judicious choice an make a problem easier (e.g. a problem involving a mass on an incline, placing x along the incline reduces the problem to a single direction).**
  3) **Write $\sum F = ma$ in each coordinate direction of interest. Keep consistent with the coordinate system (e.g. if +y is down, mg is positive).**
  4) **Integrate the equations derived in (3) once to get velocity, and again to get distance**
  5) **Apply known initial conditions from the problem statement (e.g. initial velocity, acceleration, distance) to the equations from (4)**
  6) **Use the equations from (5) with information in the problem statement to get what you want. Look for clues in the problem statement.**

Newton's Second Law

"The time rate of change in velocity (acceleration) with which an object moves is directly proportional to the magnitude of the force applied to the object and inversely proportional to the mass of the object."

Mathematically, this can be stated: $\vec{F} = m\vec{a}$ where F and a are vector quantities and mass is constant. For analyzing problems, it can be convenient to write these equations for each coordinate direction (only cartesian is given here):

$$\Sigma F_x = ma_x \ or \ \Sigma F_x = m\frac{dv_x}{dt} \ or \ \Sigma F_x = m\frac{d^2x}{dt^2} \ or \ \Sigma F_x = m\,\ddot{x} \qquad (a_x = dv_x/dt,\ v_x = dx/dt)$$

$$\Sigma F_y = ma_y \ or \ \Sigma F_y = m\frac{dv_y}{dt} \ or \ \Sigma F_y = m\frac{d^2y}{dt^2} \ or \ \Sigma F_y = m\,\ddot{y} \qquad (a_y = dv_y/dt,\ v_y = dy/dt)$$

So, if we know the force acting on an object and its mass, we can integrate to obtain velocity and integrate again to find distance. So, the degree of difficulty with integrating the above depends on the complexity of ***F***. Let's take a simple example where ***F*** equals mass under the influence of gravity, ***mg***, where we've chosen a coordinate system such that +y is down, therefore g is positive:

$$m\ g = m\frac{dv_y}{dt}$$

🡐 **Note:** coordinate system chosen such that g is in +y direction

Separating the variables, dividing both sides by m, and integrating, we get:

$$\int g dt = \int dv_y \iff gt + C_1 = v_y + C_2 \iff v_y = gt + C_1 - C_2$$

Where C_1 and C_2 are the arbitrary integration constants we learned to love in Calculus. Since they are arbitrary, there is no reason we can't combine them into a single arbitrary integration constant and call it v_{y0} , yielding:

$$v_y = g\ t + v_{y0} \qquad (1.)$$

v_{y0} = initial velocity at t = 0.

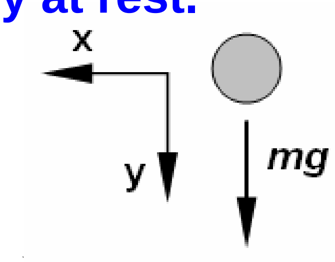
Eureka moment: did you ever wonder why in calculus the integration constants were important? In this case, it represents the initial velocity in the y direction. Why? Hint: what is v_y when t = 0? Similarly, to get y, we integrate the equation for v_y and combine integration constants:

$$v_y = g\,t + v_{y0} = dy/dt \iff \int dy = \int (gt + v_{y0})dt$$

$$y = \frac{1}{2}gt^2 + v_{y0}t + y_0 \qquad (2.)$$

y_0 = initial displacement at t = 0

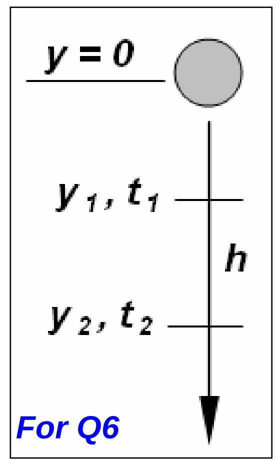
I. Free falling body – of mass m, initially at rest.



1. **FBD and coordinate system**
   Choose +*y* down (same direction as g)

2. **Equation of motion in *y***
   $\sum Fy = ma_y$ (g in +*y* direction): $(mg) = ma_y \rightarrow g = a_y$
   Integrate a_y to get v_y: $v_y = gt + v_{yo}$
   Integrate v_y to get y: $y = \frac{1}{2} gt^2 + v_{yo}t + y_o$

3. **Equation of motion in *x*:**
   $\sum F_x = ma_x$ (no force in *x* direction): $0 = a_x$
   Integrate a_x to get v_x: $v_x = v_{xo}$
   Integrate v_x to get x: $x = v_{xo}t + x_0$



4. **Apply initial conditions** (see Q2): $v_{x0} = v_{y0} = x_0 = y_0 = 0$
   $v_x = 0$ $x = 0$
   $v_y = gt$ $y = \frac{1}{2} gt^2$

Q1: why is a_y positive? Coordinate system choice (positive y is down).

Q2: why are v_x and x = 0? No force or initial velocity in x direction. Why are x_0 and y_0 equal 0? Choose coordinate origin at t = 0 position. Why are v_{x0} and v_{x0} = 0? Problem statement: 'initially at rest'

Q3: what is the velocity at $t = s_1$? Substitute s_1 into velocity equation for t to find v.

Q4: what distance does the object fall in time = s_1? Substitute s_1 into distance equation for t to find y.

Q5: How long does it take for the object to fall a distance d? Set y = d and solve for t (Hint: square root is involved).

Q6: At some point, the object falls a distance h in s seconds. What was the total distance fallen prior to this event? We need y_1 (see picture above). Using the expression for y, we can write $y_1 = \frac{1}{2} g t_1^2$ and $y_2 = \frac{1}{2} g t_2^2$. From the problem statement, we know that $y_2 - y_1 = h$ and $t_2 - t_1 = s$. These are 2 equations in 2 unknowns that can be solved by substitution for t_1 and y_1.