PHYS-2020: General Physics II
Course Lecture Notes
Section X

Dr. Donald G. Luttermoser
East Tennessee State University

Spring Semester 2004
Edition 2.0

Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 6th Edition* (2003) textbook by Serway and Faughn.

X. Interaction of Photons with Matter

A. The Classical Point of View.

1. A **system** is a collection of particles that interact among themselves via internal forces and that may interact with the world outside via external fields.

   a) To a classical physicist, a **particle** is an indivisible mass point possessing a variety of physical properties that can be measured.

      i) **Intrinsic Properties**: These don't depend on the particle's location, don't evolve with time, and aren't influenced by its physical environment (*e.g.*, rest mass and charge).

      ii) **Extrinsic Properties**: These evolve with time in response to the forces on the particle (*e.g.*, position and momentum).

   b) These measurable quantities are called **observables**.

   c) Listing values of the observables of a particle at any time $\Longrightarrow$ specify its **state**. (A **trajectory** is an equivalent way to specify a particle's state.)

   d) The *state of the system* is just the collection of the states of the particles comprising it.

2. According to classical physics, all properties, intrinsic and extrinsic, of a particle *could* be known to infinite precision $\Longrightarrow$ for instance, we could measure the precise value of both position and momentum of a particle at the same time.

3. Classical physics predicts the outcome of a measurement by calculating the **trajectory** (*i.e.*, the values of its position and momentum for all times after some initial (arbitrary) time $t_\circ$) of a particle:

$$\{\vec{r}, \vec{p}, t; t \geq t_\circ\} \equiv \text{ trajectory}, \tag{X-1}$$

where the linear momentum is, by definition,

$$\vec{p} \equiv m\,\vec{v}\ , \tag{X-2}$$

with m the mass of the particle.

a) Trajectories are *state descriptors* of Newtonian physics.

b) To study the evolution of the state represented by the trajectory in Eq. (I-1), we use Newton's Second Law:

$$ma = -\frac{\Delta \text{PE}}{\Delta r}\ , \tag{X-3}$$

where PE is the potential energy of the particle.

c) To obtain the trajectory for $t > t_\circ$, one only need to know PE and the **initial conditions** $\Longrightarrow$ the values of $\vec{r}$ and $\vec{p}$ at the initial time $t_\circ$.

d) Notice that classical physics tacitly assumes that we can measure the initial conditions without altering the motion of the particle $\Longrightarrow$ *the scheme of classical physics is based on precise specification of the position and momentum of the particle.*

4. From the discussion above, it can be seen that classical physics describes a **Determinate Universe** $\Longrightarrow$ knowing the initial conditions of the constituents of any system, however complicated, we can use Newton's Laws to predict the future.

5. If the Universe is determinate, then for every *effect* there is a *cause* $\Longrightarrow$ the **principle of causality**.

B. The Quantum Point of View.

1. The concept of a *particle* doesn't exist in the quantum world — so-called particles behave both as a particle and a wave $\Longrightarrow$ **wave-particle duality**.

a) The properties of quantum particles are not, in general, well-defined until they are measured.

b) Unlike the classical state, the quantum state is a conglomeration of several *possible* outcomes of measurements of physical properties.

c) Quantum physics can tell you only the *probability* that you will obtain one or another property.

d) An observer cannot observe a microscopic system without altering some of its properties $\Longrightarrow$ the interaction is *unavoidable*: The effect of the observer *cannot be reduced to zero*, in principle or in practice.

2. This is not just a matter of experimental uncertainties, *nature itself will not allow position and momentum to be resolved to infinite precision* $\Longrightarrow$ **Heisenberg Uncertainty Principle** (HUP):

$$\Delta x \, \Delta p_x \geq \frac{1}{2} \frac{h}{2\pi} = \frac{\hbar}{2} \, , \qquad \text{(X-4)}$$

where $h = 6.62620 \times 10^{-27}$ erg-sec $= 6.626 \times 10^{-34}$ J-sec is **Planck's Constant**.

a) Δx is the *minimum* uncertainty in the measurement of the position in the x-direction at time $t_\circ$.

b) Δp_x is the *minimum* uncertainty in the measurement of the momentum in the x-direction at time $t_\circ$.

c) Similar constraints apply to the pairs of uncertainties Δy, Δp_y and Δz, Δp_z.

d) Position and momentum are *fundamentally incompatible observables* $\Longrightarrow$ the Universe is inherently uncertain!

e) We can also write the HUP in terms of energy as

$$\Delta E \, \Delta t \geq \frac{\hbar}{2} \, . \tag{X-5}$$

f) This principle arises from geometry through a theorem known as the *Schwarz inequality of triangles.* The details of this relationship is too difficult to cover in this course. It is covered in our senior-level Quantum Physics course.

g) The HUP strikes at the very heart of classical physics: the trajectory $\Longrightarrow$ if we cannot know the position and momentum of a particle at $t_\circ$, we cannot specify the initial conditions of the particle and hence cannot calculate the trajectory.

h) Once we throw out trajectories, we can no longer use Newton's Laws, new physics must be invented!

3. Since Newtonian (*i.e.*, mechanics) and Maxwellian (*i.e.*, thermodynamics) physics describe the macroscopic world so well, physicists developing quantum mechanics demanded that when applied to macroscopic systems, the new physics must reduce to the old physics $\Longrightarrow$ this **Correspondence Principle** was coined by Neils Bohr.

4. Due to quantum mechanics probabilistic nature, only *statistical* information about *aggregates of identical systems* can be obtained. Quantum mechanics can tell us nothing about the behavior of individual systems. Moreover, the statistical information provided by quantum theory is limited to the results of measurements $\Longrightarrow$ *thou shall not make any statements that can never be verified.*

5. In the realm of the very small, various quantities (*i.e.*, energy, orbital angular momentum, spin angular momentum) are **quantized** $\Longrightarrow$ values of these parameters are not continuous, but instead, come in "jumps" or steps.

a) When we are in the realm of electrons interacting with photons (*i.e.*, distances less than 10^{-9} m), the laws of quantum mechanics describe the physics.

b) When we are in the realm of the nucleus (*i.e.*, distances less than 10^{-14} m), the laws of (nuclear) physics is described with **quantum chromodynamics**.

6. Note that in this section of the notes, we often will be using units of energy to describe masses through Einstein's equation of $E = mc^2$: 1 MeV $= 10^6$ eV $= 1.78 \times 10^{-30}$ kg. Note that the mass of the electron is 9.11×10^{-31} kg $= 0.511$ MeV.

C. Particles and Forces

1. What are the natural forces?

a) Classical physics describes forces (*i.e.*, gravity and E/M) as fields. This definition first arose with Faraday for the E/M force.

b) General relativity modified the description of the gravity field force by showing that gravity is nothing more than a curvature of space-time — the fabric of the Universe. Matter curves space-time which affects the trajectories of other matter particles traveling through that curved space-time $\Longrightarrow$ relativity states that trajectories and orbits are a result of curved space-time.

c) Modern physics describes forces as an exchange of particles, the so-called **field particles**. Each of the 4 natural forces has a field particle associated with it.

2. There are 4 natural forces (*i.e.*, those forces associated with force fields). In order of strength they are:

a) **Strong interactions**: Force that binds nucleons together — acts over a range of $\sim 10^{-15}$ m. Particles known as **hadrons** participate in the strong force. The smallest component particle of a hadron is called a **quark**. This force binds nuclei and is mediated by field particles called **gluons**.

b) **E/M interactions**: Force between charged particles which has an infinite range that falls off as $1/r^2$. This force is 100 times weaker than the strong force. This folds holds atoms and molecules together and is mediated by the **photon** field particle.

c) **Weak interactions**: These are responsible for radioactive decay (also called β-decay, since electrons are created in these reactions) of nuclei — 10^{-13} times as strong as strong interactions with a range $\ll 10^{-15}$ m. The **weakon** (also called **intermediate vector boson**) mediates this force.

d) **Gravitational interactions**: These are by far the weakest of the interactions on the microscopic scale, typically about 10^{-40} times as strong as the strong interactions on nuclear scales. Gravity is another infinite, $1/r^2$ force, except it is charge independent — as such, this force dominates all others on a cosmic scale. The (yet to be discovered) **graviton** has been proposed as the particle that mediates the gravitational force.

3. There are 2 main groups of particles that make up all matter and energy:

a) **Elementary particles**: These are particles that make up matter. They are subdivided into 3 groups:

i) **Leptons** (*light* particles) include the *electron* (e^-, $m_e = 511$ keV, 1 keV = 1000 eV, 1 eV = 1.60×10^{-19} Joules), *muon* (μ, $m_\mu = 107$ MeV), and *tau particle* (τ, $m_\tau = 1784$ MeV), each with a negative charge; their respective neutrinos: *electron neutrino* (ν_e, $m_{\nu-e} < 30$ eV), *muon neutrino* (ν_μ, $m_{\nu-\mu} < 0.5$ MeV), and *tau neutrino* (ν_τ, $m_{\nu-\tau} < 250$ MeV), each with no charge; and the antiparticles of each: e^+ (called a *positron*), $\overline{\mu}, \overline{\tau}, \overline{\nu_e}, \overline{\nu_\mu}$, and $\overline{\nu_\tau}$. These particles do **<u>not</u>** participate in the strong interactions, but do interact with the other 3 natural forces. All leptons have spin of 1/2.

ii) **Mesons** are particles of intermediate mass that are made of quark-antiquark pairs and include *pions*, *kaons*, and *η-particles*. All are unstable and decay via weak or E/M interactions. All mesons have either 0 or integer spin.

iii) **Baryons** (*heavy* particles) include the nucleons n (*neutrons* — neutral particles) and p (*protons* — positive charged) and the more massive *hyperons* (*i.e.*, Λ, Σ, Ξ, and Ω). Baryons are composed of a triplet of quarks. Each baryon has an antibaryon associated with it with a spin of either 1/2 or 3/2.

b) **Field particles**: These particles mediate the 4 natural forces as mentioned above: **gluons**, **photons**, **weakons**, and **gravitons**. These are the *energy* particles.

4. From the above list of elementary particles, there seems to be only 2 types of basic particles: *leptons* which do not obey the strong force and *quarks* which do obey the strong force. There are 6 *flavors* of leptons (as describe above). As such, it was theorized and later observed, that 6 "flavors" or *colors* of quarks (and an additional 6 antiquarks) must exist:

a) **Up** (u) quark has a rest energy of 360 MeV (1 MeV = 10^6 eV) and a charge of $+\frac{2}{3}\,e$.

b) **Down** (d) quark has a rest energy of 360 MeV and a charge of $-\frac{1}{3}\,e$.

c) **Charmed** (c) quark has a rest energy of 1500 MeV and a charge of $+\frac{2}{3}\,e$.

d) **Strange** (s) quark has a rest energy of 540 MeV and a charge of $-\frac{1}{3}\,e$.

e) **Top** (t) quark has a rest energy of 170 GeV (1 GeV = 10^9 eV) and a charge of $+\frac{2}{3}\,e$.

f) **Bottom** (b) quark has a rest energy of 5 GeV and a charge of $-\frac{1}{3}\,e$.

5. Note that a proton is composed of 2 u and a d quark and a neutron composed of an u and 2 d quarks.

6. The theory on how quarks interact with each other is called **quantum chromodynamics**. One interesting result of this theory is that quarks cannot exist in isolation, they must always travel in groups of 2 to 3 quarks.

7. There are 2 addition terms that are used to describe particles — terms that describe the *spin* of a particle:

a) In quantum mechanics, a system of identical particles 1, 2, 3, ... is described by a **wave function** (as mentioned above), which describes the spin of the particle.

b) A wave function must be either **symmetrical** (even) or **antisymmetrical** (odd) with respect to the interchange of coordinates of any pair of identical particles.

c) If symmetrical, the particles are called **bosons** and have zero or integer (*i.e.*, 0, 1, 2, 3, ...) spins.

d) If antisymmetrical, the particles are called **fermions** and have half-integer (*i.e.*, $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, ...) spins.

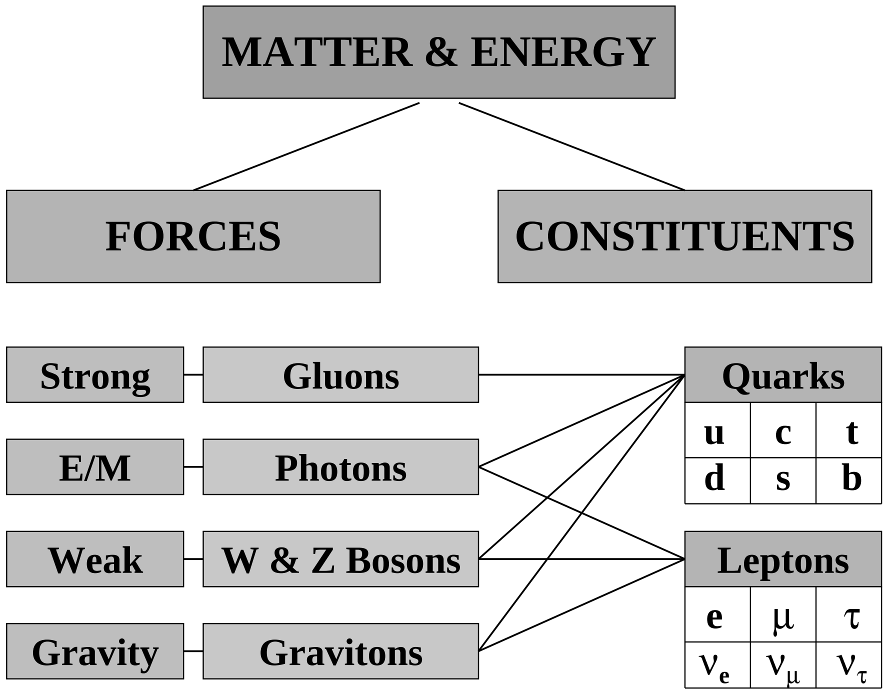
e) An antisymmetrical wave function must vanish as 2 identical particles approach each other. As a result, 2 fermions in the same quantum state exhibit a strong mutual repulsion $\Longrightarrow$ **Pauli Exclusion Principle**.

f) No such restrictions exist for bosons.

g) Leptons and baryons are fermions.

h) Mesons and field particles (*i.e.*, photons) are bosons.

8. The above description of the quantum world is called the **Standard Model of Particle Physics**. The chart below summarizes this Standard Model and Table (X-1) classifies particles based on their spins.



The Standard Model of Particle Physics

D. Atomic Physics: The Role of Quantum Numbers.

1. As mentioned above, the energy, orbital angular momentum, and spin angular momentum do not vary in a continuous way for electrons that are bound in atoms and molecules. Instead, they can only have values that are *quantized* $\Longrightarrow$ electrons can only "orbit" the nucleus of an atom in *allowed* states known as **quantum states**.

2. Each element/ion has an **electronic configuration** associated with it, which is based on the periodic table. Each e^- in that configuration has a characteristic set of quantum numbers.

a) $n \equiv$ *principal quantum number* $\Longrightarrow$ shell ID

Table X–1: Spin quantum numbers for a sample of elementary and field particles.

Common Name	**Symbol**†	**Particle Type**	**Spin (s)**	**Spin Family**
Pion	π^+	meson	0	boson
	π^0	meson	0	boson
Electron	e^-	lepton	$\frac{1}{2}$	fermion
Muon	μ^-	lepton	$\frac{1}{2}$	fermion
Neutrino	ν_e	lepton	$\frac{1}{2}$	fermion
Proton	p	baryon	$\frac{1}{2}$	fermion
Neutron	n	baryon	$\frac{1}{2}$	fermion
Gluon	G	field	1	boson
Photon	γ	field	1	boson
Weakon	W	field	1	boson
Delta	Δ^+	baryon	$\frac{3}{2}$	fermion
Graviton	g	field	2	boson

† – The superscript in the symbol corresponds to the charge of the particle: '+' = positive, '–' = negative, '0' = neutral. Symbols with no superscript are neutral, except for the proton which is positively charged, and the weakons which can have a +, –, or no electric charge.

$$\begin{array}{rcccccccc} n & = & 1 & 2 & 3 & 4 & 5 & 6 & \ldots \\ \text{shell} & : & \text{K} & \text{L} & \text{M} & \text{N} & \text{O} & \text{P} & \ldots \end{array}$$

Each shell can contain a maximum of $2n^2$ e^-s.

b) $\ell \equiv$ *orbital angular momentum quantum number* $\Longrightarrow$ subshell ID.

$$\begin{array}{rccccccccc} \ell & = & 0 & 1 & 2 & 3 & 4 & 5 & \ldots & (n-1) \\ \text{subshell} & : & \text{s} & \text{p} & \text{d} & \text{f} & \text{g} & \text{h} & \ldots & \end{array}$$

i) Each subshell can contain a max of $2(2\ell + 1)$ e^-s.

ii) The orbital angular momentum vector can have $2\ell + 1$ orientations in a magnetic field from $-\ell$ to $+\ell$:

$$-\ell \leq m_\ell \leq \ell \ .$$

c) $s \equiv$ *spin angular momentum quantum number* $\Longrightarrow$ spin direction (*i.e.*, up or down).

$$s = \frac{1}{2}$$

The spin angular momentum vector can have $2s+1$ (=2) orientations in a B-field.

$$m_s = \pm\frac{1}{2} \ .$$

d) $j \equiv$ *total angular momentum quantum number.*

$$j = \ell \pm s \ .$$

The total angular momentum vector can have $2j+1$ orientations ($-j \leq m_j \leq j$) in a B-field.

e) *Examples*:

i) An e^- with $n = 2$, $\ell = 1$, and $j = 3/2$ is denoted by $2p_{3/2}$.

ii) The lowest energy state of neutral sodium, Na I, has an e^- configuration of $1s^2\,2s^2\,2p^6\,3s$. (NOTE: the exponents indicate the number of e^-s in that subshell, no number $\equiv$ 1.) Here, the K- and L-shells are completely filled — the 3s e^- is called a **valence** e^-.

E. Photon-Matter Interactions.

1. We have seen that atoms consist of a central nucleus composed of protons, which defines the **element**, and neutrons, which defines the **isotope** of the element. A cloud of electrons surrounds this nucleus — each of these electrons exists in a *quantized* state.

2. If a photon collides with an atom, an electron can jump from one bound level to another if the energy of the photon matches the energy difference of the two states $\Longrightarrow$ a **bound-bound transition**. Once the atom is in this **excited state** (*i.e.*, an electron at a higher energy level), two different things can occur:

a) If the excited atom *collides* with another atom, the excited electron can give its energy up to the kinetic energy of the other atom, speeding it up and increasing the kinetic energy of the gas. This increase in kinetic energy means that the thermal energy of the gas increases.

i) The energy of the photon gets converted to thermal energy and is forever lost.

ii) This process is called pure **absorption**.

iii) If viewing this event from the outside, an absorption line would appear at the wavelength of the transition.

b) The electron can de-excite from the excited state in a vary short time due to either through **spontaneous emission** (which results from the HUP) or through **stimulated emission** (which results from perturbations from a nearby E/M field, *i.e.*, another photon or nearby atom).

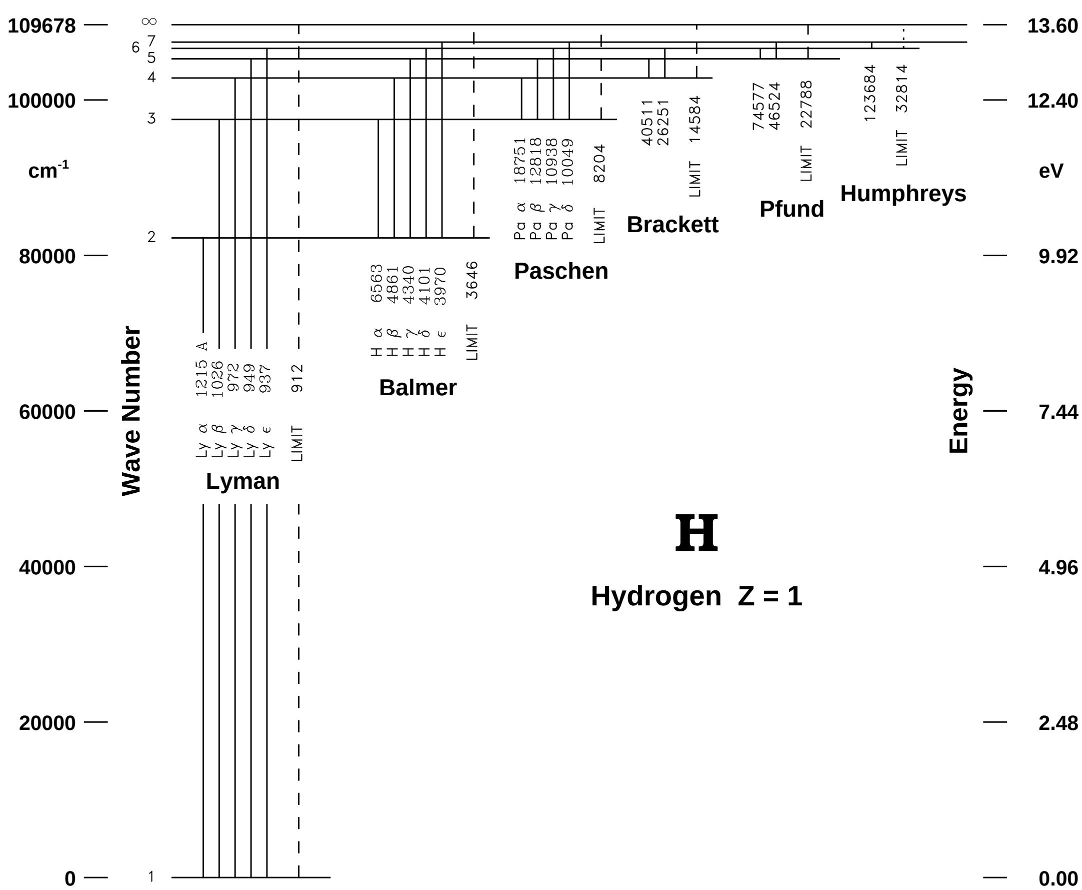
i) The photon is temporarily lost from the "beam" of light.

ii) De-excitation causes the **emission** of a photon, however, the photon can be re-emitted in any direction. As such, it might not return to the original path that it had before the interaction. Hence, this would also produce an absorption line if viewed from the outside.

iii) This process is called **scattering**.

c) This description is nothing more than Kirchhoff's laws that were discussed in the last section of the notes.

3. If a high-energy photon (one whose energy exceeds the ionization potential) interacts which an atom, the electron can be completely "ripped" off the atom in a process known as **ionization**. The reverse of this process (electron capture of an ion to produce a photon) is called **recombination**.



Example X–1. Problem 28.3 (Page 895) from the Serway & Faughn textbook: The "size" of the *atom* in Rutherford's model (which has not been discussed) is about 1.0×10^{-10} m. (a) Determine the attractive electrical force between an electron and a proton separated by this distance. (b) Determine (in eV) the electrical potential energy of the atom.

Solution (a):

From Coulomb's law,

$$
\begin{aligned}
|F_e| &= \frac{k_e|q_1||q_2|}{r^2} = \frac{(8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.0 \times 10^{-10} \text{ m})^2} \\
&= \boxed{2.3 \times 10^{-8} \text{ N}\ .}
\end{aligned}
$$

Solution (b):

The electrical potential energy is

$$
\begin{aligned}
V &= \frac{k_e q_1 q_2}{r} \\
&= \frac{(8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)(-1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{1.0 \times 10^{-10} \text{ m}} \\
&= -2.3 \times 10^{-18} \text{ J} \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = \boxed{-14 \text{ eV}\ .}
\end{aligned}
$$

F. The Bohr Model of Hydrogen.

1. Work that lead to an understanding of the spectrum of the hydrogen atom took place at the end of the 19th and beginning of the 20th century. As such, the work described here is presented in the cgs unit system since those are the units that were being used in physics at the time.

2. Rydberg (1890), Ritz (1908), Planck (1910), and Bohr (1913) were all responsible for developing the theory of the spectrum of the H atom. A transition from an upper level m to a lower level n will radiate a photon at frequency

$$\nu_{mn} = c\, R_A\, Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) , \qquad \text{(X-6)}$$

where the velocity of light, $c = 2.997925 \times 10^{10}$ cm/s, Z is the *effective* charge of the nucleus ($Z_H = 1, Z_{He} = 2$, etc.), and the atomic Rydberg constant, R_A, is given by

$$R_A = R_\infty \left(1 + \frac{m_e}{M_A} \right)^{-1} . \qquad \text{(X-7)}$$

a) The Rydberg constant for an infinite mass is

$$\begin{aligned} R_\infty &= \frac{2\pi^2\, m_e\, e^4}{c\, h^3} = 109,737.31 \text{ cm}^{-1} \\ &= 1.0973731 \times 10^7 \text{ m}^{-1}, \end{aligned} \qquad \text{(X-8)}$$

where $e = 4.80325 \times 10^{-10}$ esu is the electron charge in cgs units.

b) In atomic mass units (amu), the electron mass is $m_e = 5.48597 \times 10^{-4}$ amu whereas the atomic mass, M_A, can be found on a periodic table (see also Table I-2).

c) Eq. (X-6) can also be expressed in wavelengths (vacuum) by the following

$$\frac{1}{\lambda_{mn}} = R_A\, Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) . \qquad \text{(X-9)}$$

3. Lines that originate from the same level in a hydrogen-like atom/ion are said to belong to the same series. Transitions out of (or into) the ground state ($n = 1$) are lines of the **Lyman series**, $n = 2$ corresponds to the **Balmer series**, and $n = 3$, the **Paschen series**.

Table X–2: Atomic Masses and Rydberg Constants

Atom	**Atomic Mass, $\mathbf{M_A}$ (amu)**	**Rydberg Constant, $\mathbf{R_A}$ ($\mathbf{cm^{-1}}$)**
Hydrogen, ^{1}H	1.007825	109,677.6
Helium, ^{4}He	4.002603	109,722.3
Carbon, ^{12}C	12.000000	109,732.3
Nitrogen, ^{14}N	14.003074	109,733.0
Oxygen, ^{16}O	15.994915	109,733.5
Neon, ^{20}Ne	19.992440	109,734.3

4. For each series, the transition with the longest wavelength is called the *alpha* (α) transition, the next blueward line from α is the β line followed by the γ line, etc.

a) Lyman α is the $1 \leftrightarrow 2$ transition, Lyman β is the $1 \leftrightarrow 3$ transition, Lyman γ is the $1 \leftrightarrow 4$ transition, etc.

b) Balmer or Hα is the $2 \leftrightarrow 3$ transition, Hβ is the $2 \leftrightarrow 4$ transition, Hγ is the $2 \leftrightarrow 5$ transition, etc.

5. Lines that go into or come out of the ground state are referred to as **resonance lines**.

6. For one e^- atoms (*i.e.*, hydrogen-like: H I, He II, C VI, Fe XXVI, etc. $\Longrightarrow$ in astrophysics, ionization stages are labeled with roman numerals: I = neutral, II = singly ionized, etc.), the principal (n) levels have energies of

$$E_n = -\frac{2\,\pi^2\, m\, e^4\, Z^2}{n^2\, h^2} = (-13.6 \text{ eV})\,\frac{Z^2}{n^2}\ , \qquad \text{(X-10)}$$

where Z = charge of the nucleus.

a) Negative energies $\Longrightarrow$ bound states
Positive energies $\Longrightarrow$ free states
Ionization limit ($n \rightarrow \infty$) in Eq. (X-10) has $E = 0$.

b) In astronomical spectroscopy, the ground state is defined as zero potential (*i.e.*, $E_1 = 0$) and atomic states are displayed in terms of *energy level diagrams* (as shown in the figure on Page X-14), where the energy levels are determined by

$$E_n = 13.6\ Z^2 \left(1 - \frac{1}{n^2}\right)\ \mathrm{eV}\ . \qquad \text{(X-11)}$$

$n \to \infty$ defines the **ionization potential** (IP) of the atom (or ion), so that for H: IP = 13.6 eV, for He II: IP = 54.4 eV, etc.

c) The lowest energy state ($E = 0$) is called the **ground state**. States above the ground are said to be **excited**.

d) The ionization "edge" of the series in a spectrum, the **series limit**, is found by letting the lower quantum number (n) in Eq. (X-9) going to ∞ or by rewriting the equations above in terms of wavelength. Using Eq. (X-9) and relabeling level m as level n (for consistency with Eqs. X-10 and X-11) we get

$$\frac{1}{\lambda_n} = \frac{R_A\,Z^2}{n^2} \qquad \text{(X-12)}$$

$\Longrightarrow$ Lyman edge: 912 Å = 91.2 nm, Balmer edge: 3646 Å = 364.6 nm.

7. Bohr also determined that the distance that a state is from the nucleus (*i.e.*, a proton in the case of hydrogen) can be determined in a semi-classical approach (quantum mechanics had not yet been invented when Bohr did this) by setting the angular momentum L of a mass in a circular orbit to an integer multiple of $\hbar = h/2\pi$:

$$L_n = m_e v_n r_n = n\hbar\ . \qquad \text{(X-13)}$$

a) Using the energy equation (Eq. X-10) to determine v_n, Bohr derived the following formula for the radii of each quantum orbit = **state**:

$$r_n = \frac{n^2\hbar^2}{m_e k_{\rm B} e^2} = n^2 a_\circ = n^2(0.0529 \text{ nm}) \ . \qquad \text{(X-14)}$$

b) The radius of the ground state is $a_\circ$ which is called the **Bohr radius** is given by

$$a_\circ = \frac{\hbar^2}{m_e k_{\rm B} e^2} \ . \qquad \text{(X-15)}$$

c) For a hydrogen-like ion, the radii of each quantum orbit is

$$r_n = \frac{n^2\hbar^2}{Z m_e k_{\rm B} e^2} = \frac{n^2 a_\circ}{Z} = \frac{n^2(0.0529 \text{ nm})}{Z} \ . \qquad \text{(X-16)}$$

Example X–2. Problem 28.8 (Page 895) from the Serway & Faughn textbook: For a hydrogen atom in its ground state, use the Bohr model to compute (a) the orbital speed of an electron, (b) the kinetic energy of an electron, and (c) the electrical potential energy of the atom.

Solution (a):

With electrical force supplying the centripetal acceleration,

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2} \ , \quad \text{giving} \quad v_n = \sqrt{\frac{k_e e^2}{m_e r_n}} \ ,$$

where $r_n = n^2 a_\circ = n^2(0.0529 \text{ nm})$. Thus,

$$\begin{aligned} v_1 &= \sqrt{\frac{k_e e^2}{m_e r_1}} \\ &= \sqrt{\frac{(8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(0.0529 \times 10^{-9} \text{ m})}} \\ &= \boxed{2.19 \times 10^6 \text{ m/s} \ .} \end{aligned}$$

Solution (b):

The kinetic energy is then

$$\begin{aligned}
\mathrm{KE}_1 &= \frac{1}{2}m_e v_1^2 = \frac{1}{2}m_e\left(\frac{k_e e^2}{m_e r_1}\right)^2 = \frac{k_e e^2}{2r_1} \\
&= \frac{(8.99\times10^{9}\ \mathrm{N{\cdot}m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2}{2(0.0529\times10^{-9}\ \mathrm{m})} \\
&= 2.18\times10^{-18}\ \mathrm{J}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{13.6\ \mathrm{eV}\ .}
\end{aligned}$$

Solution (c):

The potential energy is

$$\begin{aligned}
\mathrm{PE}_1 &= \frac{k_e(-e)e}{r_1} \\
&= \frac{(8.99\times10^{9}\ \mathrm{N{\cdot}m^2/C^2})(-1.60\times10^{-19}\ \mathrm{C})(1.60\times10^{-19}\ \mathrm{C})}{(0.0529\times10^{-9}\ \mathrm{m})} \\
&= -4.35\times10^{-18}\ \mathrm{J}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{-27.2\ \mathrm{eV}\ .}
\end{aligned}$$

Example X–3. **Problem 28.20 (Page 896) from the Serway & Faughn textbook**: An electron is in the first Bohr orbit of hydrogen. Find (a) the speed of the electron, (b) the time required for the electron to circle the nucleus, and (c) the current in amperes corresponding to the motion of the electron.

Solution (a):

Using Eq. (X-13) for the angular momentum, we can solve of v_n and setting $n = 1$ for the ground state, we get

$$\begin{aligned}
m_e v_n r_n &= n\hbar \\
v_n &= \frac{n\hbar}{m_e r_n}
\end{aligned}$$

$$\begin{aligned} v_1 &= \frac{\hbar}{m_e r_1} = \frac{h}{2\pi m_e a_\circ} \\ &= \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(9.11\times10^{-31}\ \text{kg})(0.0529\times10^{-9}\ \text{m})} \\ &= \boxed{2.19\times10^{6}\ \text{m/s}\ .} \end{aligned}$$

Solution (b):

This velocity just represents the path taken per one cycle. Since the path is the circumference of a circle, $C = 2\pi r$, the amount of time it will take to complete one orbit (*i.e.*, cycle) is

$$\begin{aligned} v_1 &= \frac{C_1}{\Delta t} = \frac{2\pi r_1}{\Delta t} \\ \Delta t &= \frac{2\pi r_1}{v_1} = \frac{2\pi(0.0529\times10^{-9}\ \text{m})}{2.19\times10^{6}\ \text{m/s}} \\ &= \boxed{1.52\times10^{-16}\ \text{s}\ .} \end{aligned}$$

Solution (c):

We just need to use the definition of current to determine its value:

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{|e|}{\Delta t} \\ &= \frac{1.60\times10^{-19}\ \text{C}}{1.52\times10^{-16}\ \text{s}} \\ &= 1.05\times10^{-3}\ \text{A} = \boxed{1.05\ \text{mA}\ .} \end{aligned}$$

Example X–4. **Problem 28.27 (Page 896) from the Serway & Faughn textbook**: (a) Find the energy of the electron in the ground state of doubly ionized lithium, which has an atomic number of $Z = 3$. (b) Find the radius of its ground state orbit.

Solution (a):

For this we use Eq. (X-10) with $n = 1$ and $Z = 3$:

$$\begin{aligned} E_1 &= (-13.6 \text{ eV}) \frac{Z^2}{n^2} = (-13.6 \text{ eV}) \frac{3^2}{1^2} \\ &= \boxed{-122 \text{ eV} .} \end{aligned}$$

Solution (b):

For this we use Eq. (X-16) with $n = 1$ and $Z = 3$:

$$r_1 = \frac{n^2 a_\circ}{Z} = \frac{1^2(0.0529 \text{ m})}{3} = \boxed{1.76 \times 10^{-11} \text{ m} .}$$

Example X–5. **Problem 28.10 (Page 895) from the Serway & Faughn textbook**: A photon is emitted as a hydrogen atom undergoes a transition from the $n = 6$ to the $n = 2$ state. Calculate (a) the energy, (b) the wavelength, and (c) the frequency. (Added question: (d) To which series does series in the hydrogen spectrum does this photon belong?)

Solution (b):

For this question, the solution to (a) can easily be derived from the solution to (b). As such, we will do this solution first. Using Eq. (X-9) with $Z = 1$, $m = 6$, and $n = 2$, and making use of the data in Table X-2, we get

$$\begin{aligned} \frac{1}{\lambda} &= R_A Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \\ &= R_H (1)^2 \left(\frac{1}{2^2} - \frac{1}{6^2} \right) \\ &= (1.096776 \times 10^5 \text{ cm}^{-1}) \left(\frac{1}{4} - \frac{1}{36} \right) = 2.43728 \times 10^4 \text{ cm}^{-1} \\ \lambda &= 4.1029 \times 10^{-5} \text{ cm} \left(\frac{1 \text{ nm}}{10^{-7} \text{ cm}} \right) = \boxed{410.29 \text{ nm} .} \end{aligned}$$

Solution (a):

Now we can easily answer part (a) using Eq. (IX-5):

$$\begin{aligned} E &= \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \mathrm{J{\cdot}s})(3.00 \times 10^{8}\ \mathrm{m/s})}{4.1029 \times 10^{-7}\ \mathrm{m}} \\ &= 4.85 \times 10^{-19}\ \mathrm{J} = \boxed{3.03\ \mathrm{eV}\ .} \end{aligned}$$

Solution (c):

For this we use Eq. (IX-4):

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \mathrm{m/s}}{4.1029 \times 10^{-7}\ \mathrm{m}} = \boxed{7.31 \times 10^{14}\ \mathrm{Hz}\ .}$$

Solution (d):

Since the electron ends up on the second level ($n = 2$), this emission line belongs to the hydrogen $\boxed{\textbf{Balmer series.}}$
