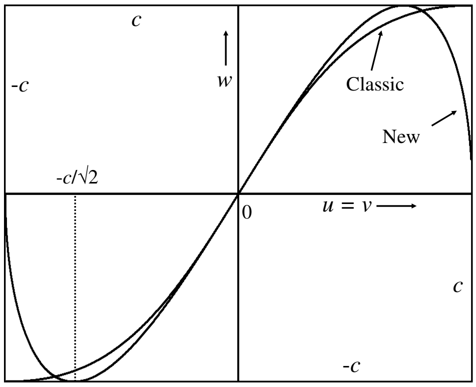

Figure 6: Classical [Eq. (18)] and newly derived graph [Eq. (19)] for relativistic addition of velocities plotted together for $u = v$.

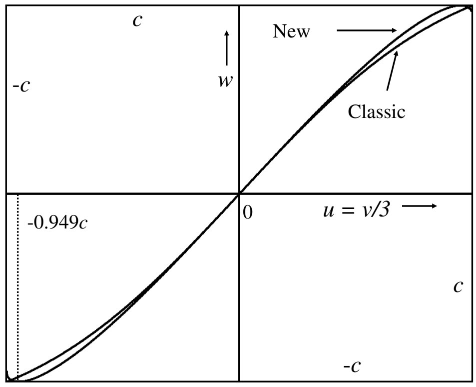

Figure 7: Classical [Eq. (18)] and newly derived graph [Eq. (19)] for relativistic addition of velocities plotted together for $u = v/3$.

anti-particles, running 'backwards in time'.).

The situation where u equals v gives the maximum possible deviation relative to the classical graph. Other ratios between u and v give (much) smaller deviations and the tops of Eq. (19) will shift outwards towards c as can be seen in Fig. 7 where the ratio between u and v equals 3:1. At a ratio 10:1 both plots are practically identical. Virtually all practical situations that require the velocity addition formula to be used exist under such circumstances, which indicates that a deviation from the classical graph is likely to remain unnoticed.

3. Some interpretations of Fizeau's experiment give rise to doubt concerning the correctness of Eq. (18). If Eq. (19) is used in the analysis of Fizeau's experiment done by Renshaw [7], it yields better results than Eq. (18), although still not within the margins as claimed by Michelson.

   The vast majority of experimental set-ups that are aimed at verification of relativity theory are using two reference frames. These experiments are not suitable for the verification of the velocity addition formula. One would have to use a set-up with three reference frames. At speeds on the order of 10^4 m/s the difference in resulting values between Eqs. (18) and (19) is on the order of 10^{-5} m/s, which might be noticeable using adequately accurate measuring devices.

A hypothetical case will now be used to show that Eq. (19) does not necessarily lead to causality conflicts as a result of the negative time-speeds that can occur.

A spaceship travels relative to Earth at speed $v_s = 0.9c$ and heads toward an asteroid that is at rest relative to Earth. The ship launches a missile at the asteroid at $v_m = 0.9c$ relative to the ship. An observer on the ship watches the missile destroy the asteroid. According to Eq. (19), an observer on Earth would see the missile traveling at only $0.7846c$ so the missile's spatial speed is *lower* than that of the spaceship. It seems therefor that this observer would see the ship hit the asteroid before the missile.

The explanation of this paradox can be found in the comparison of the proper times of all objects involved. We call the proper time for the spaceship τ_s and for the missile τ_m. For simplicity we set the space-time event of the launch at $t = \tau_m = \tau_s = 0$ and the distance between the spaceship and the asteroid at that moment at 0.9 light second (as measured by the observer on Earth).

The observer on Earth calculates time-coordinates of the impact (against the asteroid) using his own time t for the spaceship: $t_s = 1$s;

and for the missile: $t_m = 0.9/0.7846 = 1.147$s, so it seems as if the spaceship reaches the asteroid first. In 4D Euclidean space-time however the observer measures the time-speed χ_s of the spaceship as: $\chi_s = \sqrt{c^2 - v_s^2} = \sqrt{c^2 - (0.9c)^2} = 0.4359c$.

According to this observer the absolute value of the timespeed χ_m of the missile is $\chi_m = \sqrt{c^2 - (0.7846c)^2} = 0.62c$, but from the circle diagram (Fig. 3) it shows that we must now take the negative root so its value is $\chi_m = -0.62c$. Note that the cyclic nature of γ now also implies that in this situation γ has a negative value in $\tau_m = t_m/\gamma = t_m\chi_m/c$ for the missile.

We calculate the proper times at the moment of impact according to the observer on Earth for the spaceship: $\tau_s = t_s\chi_s/c = 0.4359$s; and for the missile: $\tau_m = 1.147(-0.62) = -0.7111$s.

In proper time the missile hits the asteroid before the spaceship does despite its lower spatial speed. Causality is therefor not violated. The missile runs backwards in proper time.

5 Relativistic Doppler Effect

Using the identity $\chi = \sqrt{c^2 - v^2}$ for the time-speed variable in the wavelength equation for the relativistic Doppler effect

$$\lambda' = \lambda_0 \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{20}$$

simplifies this expression to

$$\lambda' = \lambda_0 (c + v)/\chi \tag{21}$$

It is possible to identify the individual contributions of the factors v and χ to the total Doppler effect by considering $\chi = c$ (which isolates the effect of the spatial speed) and $v = 0$ (which isolates the effect of the time-speed).

Setting $\chi = c$ results in:

$$\lambda'_v = \lambda_0 (1 + v/c) \tag{22}$$

which is the regular equation for the acoustic Doppler effect with moving source and stationary receiver. Setting $v = 0$ results in:

$$\lambda'_\chi = \lambda_0 c/\chi \tag{23}$$

which simply makes the photon's frequency proportional to the time-speed of the emitting particle.

The relativistic Doppler effect can thus be interpreted as a combination of the normal 'acoustic' Doppler effect in space and a frequency shift that results from the lower time-speed.

6 Mass, Energy and Momentum

Figure 8 depicts a moving object with spatial velocity $\mathbf{V}$ of magnitude v, as measured by an observer at point L, at rest.

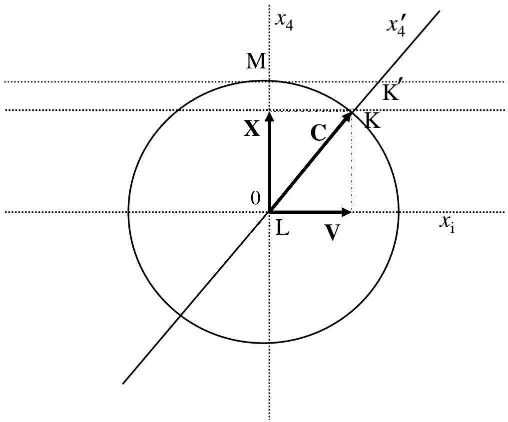

Figure 8: 4D velocity of magnitude c in x'_4 of an object at L. An observer at rest at L has velocity of magnitude c in x_4.

The vector sum of spatial and time-velocities reflects the four-velocities of the observer (along x_4) and the moving object (along x'_4). It follows naturally that the Lorentz invariant m_0c (m_0 is the rest mass) in the moving object A can be decomposed in

$$m_0^2c^2 = m_0^2\chi^2 + m_0^2v^2 \tag{24}$$

which, using the identities $E = \gamma m_0 c^2$ and $p = \gamma m_0 v$, is equivalent to the classical equation

$$E^2/c^2 = m_0^2c^2 + p^2 \tag{25}$$

E being the total energy and p being the spatial momentum.

The components in the right part of Eq. (24) cannot simply be interpreted as, respectively, the object's momenta in the time dimension and the spatial dimension of the rest frame of the observer.

There is an obvious problem in the fact that the factor γ is involved in the expressions for E and p. If we multiply the factor γ^2 into all three elements of Eq. (24) we get:

$$\gamma^2 m_0^2 c^2 = \gamma^2 m_0^2 \chi^2 + \gamma^2 m_0^2 v^2 \tag{26}$$

which describes triangle LK'M (if m_0 is set to 1). This alternative form for Eq. (24) immediately shows the meaning of its components. They now correspond one to one with the components in Eq. (25): $\gamma m_0 c = E/c$, $\gamma m_0 \chi = m_0 c$, $\gamma m_0 v = p$. The factor $\gamma m_0 c$ is however not invariant under rotations in SO(4), while $m_0 c$ is. [Note that although $m_0 c$ is indeed Lorentz invariant from the perspective of the observer, its physical meaning in its own rest frame is the moving object's time-momentum. The same invariant value can be found in the rest frame of the observer (see also Fig. 9) but should then be read as $\gamma m_0 \chi$.] The Lagrangian formalism for this situation has been worked out by Montanus in [2]. The reader is therefore referred to this source for the detailed derivation. The generic principles used for such 5D situations (or more generally 4D with the addition of an extra parameter to keep track of the progress of the object along its world-line) appear in Goldstein [8]. The latter however uses the classical indefinite Minkowski metric as a basis for the development of the relativistic Lagrangian Λ where Montanus uses a positive definite metric like in this article. A short overview of the main equations is given here.

In agreement with classical mechanics it is assumed that the variation according to Hamilton's principle:

$$\delta I = \int_{x_{5(1)}}^{x_{5(2)}} \Lambda(x_\mu, u_\mu) dx_5 \tag{27}$$

is an extremum, where $u_\mu = dx_\mu / dx_5$. The corresponding Euler-Lagrange equations of motion are:

$$\frac{\partial \Lambda}{\partial x_\mu} - \frac{d}{dx_5}(\partial \Lambda / \partial u_\mu) = 0 \tag{28}$$

leading to a possible relativistic Lagrangian for a free object in the absence of a forcefield (so the potential energy equals zero):

$$\Lambda = m_0 u_\mu u^\mu \tag{29}$$

which equals, as a result of the universal velocity magnitude c for the free particle in 4D space-time:

$$\Lambda = m_0 c^2 \tag{30}$$

The latter is to be interpreted as the 'kinetic energy' of the particle in four dimensions, which is a fundamentally different concept than kinetic energy in three dimensions. It corresponds to the total energy of a particle at rest. Other solutions for Λ are possible but the essential element is that any solution is a constant in 4D space-time.

The relativistic Lagrangian Λ shows that the factor γ in Eq. (26) must be a result of our confinement to a 3D subspace of 4D space-time. In order to maintain conservation laws for energy and momentum, while only being able to measure their 'projections' to our 3D space, the factor γ is an artificial necessity. It vanishes for a hypothetical observer with full 4D observational skills, who measures the object's speed and energy as constants.

7 Transformation of Energy and Momentum

The generic transformation equations for energy and momentum depend indirectly on the equation for relativistic addition of velocities. Because a new one replaces this equation, it is necessary to rework the transformation equations for energy and momentum as well.

Figure 9 depicts an object moving with velocity **W** of magnitude w relative to frame x and velocity **U** of magnitude u relative to frame x'.

(please refer also to Fig. 3 and the definitions given there)

- $E = \gamma(w) m_0 c^2$ is the energy of an object that moves with velocity **W** of magnitude w relative to frame x and measured in frame x.
- $E' = \gamma(u) m_0 c^2$ is the energy of that same object moving with velocity **U** of magnitude u relative to frame x' and measured from frame x'.
- Frame x' moves with velocity **V** of magnitude v relative to frame x.
- $\gamma(u) = 1 \Big/ \sqrt{1 - u^2/c^2}$

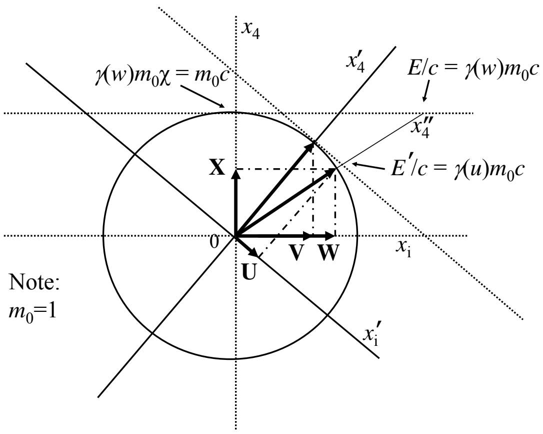

Figure 9: Generic transformation of energy and momentum in three reference frames with rotated dimensional axes.

- $\gamma(v) = 1 \Big/ \sqrt{1 - v^2/c^2}$
- $\gamma(w) = 1 \Big/ \sqrt{1 - w^2/c^2}$

For energy this leads to a generic transformation equation

$$E/E' = \gamma(w)/\gamma(u) \tag{31}$$

which can be written in different forms using Eq. (19). With $u = 0$ this reduces to the classical form:

$$E/E' = \gamma(v) \tag{32}$$

For momentum a generic transformation equation is

$$p/p' = wE/uE' \tag{33}$$

where:

- $p' = \gamma(u)m_0u$ is the momentum of the object as measured from frame x'.
- $p = \gamma(w)m_0w$ is the momentum of the object as measured from frame x.

8 Euclidean Four-Vectors

The traditional Minkowski line element with metric $(+1, -1, -1, -1)$ is:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{34}$$

where $ds = cd\tau$. Four-vectors with the Euclidean metric $(+1, +1, +1, +1)$ as used in the previous Sections use the 4D velocity of the moving object and 4D Euclidean distances as invariants, which is in fact the essence of Eq. (2):

$$c^2 = v_1^2 + v_2^2 + v_3^2 + \chi^2 \tag{35}$$

Multiplication with $dt^2 = dx_5^2$ yields (recall that $\chi = cd\tau/dt$):

$$c^2dt^2 = dx_1^2 + dx_2^2 + dx_3^2 + c^2d\tau^2 \tag{36}$$

where the factors $c^2d\tau^2$ and c^2dt^2 from Eq. (34) have switched roles.

The Euclidean metric thus gives rise to four-vectors for position, velocity and energy/momentum:

Euclidean	Minkowskian
$(x_1, x_2, x_3, c\tau)$	(x_1, x_2, x_3, ct)
(v_1, v_2, v_3, χ)	$\gamma(v_1, v_2, v_3, c)$
$(m_0v_1, m_0v_2, m_0v_3, m_0\chi)$	$(p_1, p_2, p_3, E/c)$

Equation (36) is not really new. It is merely Eq. (34) written in a different form, with as a main input the definition of χ, being the time-speed of an object as measured by an observer at rest, which has three effects:

- It creates a new invariant c, being the universal magnitude of the 4D velocity of an object.
- It provides a Euclidean basis for the definition of vectors in the direction of the time dimension.
- It enables these new vectors to be summed with existing vectors in the spatial dimensions.

In general, the new Euclidean four-vectors can be derived from the Minkowski four-vectors by using the time component in the Minkowski four-vector as the invariant (the vector sum) for the new four-vector. It is essentially doing Pythagoras "the other way around", *i.e.*, calculating the hypotenuse from the rectangular sides, instead of calculating a rectangular side from the hypotenuse and the other rectangular side (refer to [9] for a detailed treatment of Minkowski and Euclidean four-vectors).

References

[1] H. Montanus, "Proper-Time Formulation of Relativistic Dynamics", Foundations of Physics **31** (9) 1357-1400 (2001).

[2] H. Montanus, "Special Relativity in an Absolute Euclidean Space-Time", Physics Essays **4** (3) 350-356 (1991).

[3] A. Gersten, "Euclidean special relativity", Foundations of Physics **33** (8) 1237-1251 (2003).

[4] Jose B. Almeida, "An Alternative to Minkowski Space-Time" (arXiv:gr-qc/0104029 v2, 10 Jun 2001).

[5] Robert d'E Atkinson, "General Relativity in Euclidean Terms", Royal Society of London Proceedings Series A **272** (1348) 60-78 (1963).

[6] Brian R. Greene, **The elegant universe**, page 391 note 5 (W.W. Norton & Company New York-London, 1999).

[7] C. Renshaw, "The Experiment of Fizeau as a Test of Relativistic Simultaneity", available at: renshaw.teleinc.com.

[8] H. Goldstein, **Classical Mechanics**, Second edition, Chapter 7-9 (Addison Wesley, 1980).

[9] R.F.J. van Linden "Minkowski versus Euclidean 4-vectors", available at: www.euclideanrelativity.com.