Subject

$$\frac{\partial \phi(x,y,z,t)}{\partial t} = D \nabla \phi(x,y,z,t)$$

$$= D \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi(x,y,z,t)$$

$$\frac{\partial \phi}{\partial t} = D \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi$$

$$\phi(x,y,z,t) = \alpha(x) \beta(y) \gamma(z) \delta(t)$$

$$\underbrace{\frac{1}{\delta} \frac{\partial \delta}{\partial t}}_{\lambda} = D \left[\underbrace{\frac{1}{\alpha} \frac{\partial^2 \alpha}{\partial x^2}}_{\lambda x} + \underbrace{\frac{1}{\beta} \frac{\partial^2 \beta}{\partial y^2}}_{\lambda y} + \underbrace{\frac{1}{\gamma} \frac{\partial^2 \gamma}{\partial z^2}}_{\lambda z} \right]$$

$$\frac{\partial \delta}{\partial t} = \lambda \delta \Rightarrow \delta = A e^{\lambda t}$$

$$\frac{\partial^2 \alpha}{\partial x^2} = \lambda_x \alpha \qquad \alpha(x) = B\cos(hx) + C\sin(hx)$$

$$\alpha(-r(z)) = \alpha(+r(z)) = 0$$

$$\Rightarrow C = 0 \qquad h = \frac{\pi}{2r(z)} \Rightarrow \alpha = B\cos\left(\frac{\pi}{2r} x\right)$$

Similarly $\beta(y) = C\cos\left(\frac{\pi}{2r}y\right)$

$$\frac{\partial^2 \gamma(z)}{\partial z^2} = \lambda_z \gamma \qquad \gamma = D\cos(lz) + E\sin(lz)$$

$$\gamma(0) = C_0 \Rightarrow D = C_0$$

$$\gamma(H+R) = 0 \Rightarrow E = 0 \quad l = \frac{\pi}{2(H+R)}$$

$$\Rightarrow \gamma = C_0 \cos\left(\frac{\pi}{2(H+R)} z\right)$$

$$\alpha(x) = B\cos\left(\frac{\pi}{2r}x\right) \rightarrow \frac{\partial^2\alpha}{\partial x^2} = \lambda_x \alpha$$

$$\Rightarrow \lambda_x = -\frac{\pi^2}{4r^2}$$

Similarly $\beta(x) = C\cos\left(\frac{\pi}{2r}y\right) \rightarrow \frac{\partial^3\beta}{\partial x^2} = \lambda_y \beta$

$$\Rightarrow \lambda_y = -\frac{\pi^2}{4r^2} \qquad \lambda_z = -\frac{\pi^2}{4(H+R)}$$

$$\Rightarrow \lambda = D(\lambda_x + x_y + \lambda_z) < 0$$