The equation of motion in the BCS model is

$$i\frac{\partial}{\partial t}\psi_\alpha(x) + \left(\frac{\Delta}{2m} + \mu\right)\psi_\alpha(x) + g\psi_\beta^\dagger(x)\psi_\beta(x)\psi_\alpha(x) = 0 \;, \tag{2.86}$$

a rather complicated nonlinear equation. The BCS approximation to its solution occurs upon making the following substitution into (2.86):

$$\begin{aligned} \psi_\beta(x)\psi_\alpha(x) &\longrightarrow \langle 0|\psi_\beta(x)\psi_\alpha(x)|0\rangle \\ &= \lim_{x\to y}\langle 0|T\left(\psi_\beta(x)\psi_\alpha(y)\right)|0\rangle \equiv iF(0)\epsilon_{\beta\alpha} \;, \end{aligned} \tag{2.87}$$

where $|0\rangle$ is the ground state of the model, and $F(x-y)\epsilon_{\beta\alpha}$ is the anomalous part of the fermion propagator (compare with Eq.(2.81)).

This substitution reflects the essence of the dynamics in the BCS model. The composite operator $\psi_\beta(x)\psi_\alpha(x)$ describes a condensate of electrons. The substitution corresponds to the approximation in which this operator is treated as the c-number $iF(0)\epsilon_{\alpha\beta}$, i.e. it is assumed that its classical part dominates. One can see a striking similarity of the BCS approximation with the approximation used before in the problem of a nonideal Bose gas (see Sec.2.5). In that case the condensate was described by the elementary field $\varphi(x)$; the condensate led to the spontaneous breakdown of particle number conservation. In the BCS model the condensate described by the composite operator $\psi_\beta(x)\psi_\alpha(x)$ leads to the spontaneous breakdown of the $U(1)$ symmetry connected with the conservation of the electric charge.

With this substitution, equation (2.86) simplifies to become the equation

$$i\frac{\partial}{\partial t}\psi_\alpha(x) + \left(\frac{\Delta}{2m} + \mu\right)\psi_\alpha(x) + M_d\epsilon_{\alpha\gamma}\psi_\gamma^\dagger(x) = 0 \;, \tag{2.88}$$

where $M_d \equiv -igF(0)$ can be considered as a *dynamical* Majorana mass. Indeed, if we replace M_d with M , this equation exactly coincides with that of the Fermi gas with a Majorana mass. Therefore we can use freely the results of the preceding section.

In particular the fermion propagator (see Eqs. (2.80) and (2.84)) is given by

$$G(\omega, \mathbf{p}) = \frac{\Gamma_0\omega + \epsilon(p) + |M_d|\Gamma_2}{\omega^2 - |M_d|^2 - \epsilon^2(p) + i0}\Gamma_0 \;, \tag{2.89}$$