Übungen zu "Mathematische Methoden der Physik II"

Problem set 2 **(starting 14.10.2013)**

Problem 4: Consider

$$y'' + a_1 y' + a_2 y = f(x) \tag{1}$$

Write down the general solution of the homogeneous equation in the form $y_{\text{hom}} = c_1 y_1(x) + c_2 y_2(x)$, and determine $y_{1,2}(x)$ (you may assume that $a_1^2 \neq 4a_2$).
Now we seek a solution of the inhomogeneous equation via "variation of the constant",

$$y_{\text{spec}} = c_1(x) y_1(x) + c_2(x) y_2(x) \tag{2}$$

imposing also $c_1' y_1(x) + c_2' y_2(x) = 0$.
Show that such a solution is obtained by

$$c_1(x) = -\int \frac{y_2 f}{W} dx', \qquad c_2(x) = \int \frac{y_1 f}{W} dx' \tag{3}$$

where $W(x)$ is the Wronskian.
In particular, find all (real-valued) solutions of $y'' + y = \sinh(x)$.

Problem 5:

Find a complete set of solutions for

$$y'' - z y'(z) + 2y(z) = 0. \tag{4}$$

Hint: Expand the solution in a Taylor series around $z = 0$. Find one elementary solution explicitly, and the other in terms of an explicit integral using the method in section 1.1.5.

Problem 6:

Consider the (real-valued) differential equation

$$y'' + k^2 y = 0$$

in the interval $x \in [0, L]$ with the boundary conditions $y(0) = 0$ and $y'(L) = 0$.
For which $k \in \mathbb{R}$ does this equations admit non-trivial (=non-vanishing) solutions, for the given boundary conditions? Give all such solutions for all admissible values of k.