Subject ·

Übung 5) $f(z)$ $g(z)$

$$y'' - \underbrace{z}\, y'(z) + \underbrace{2}\, y(z) = 0$$

$z_0 = 0 \rightarrow f(z)$ and $g(z)$ are analytic in $z_0 = 0$

$$\Rightarrow y(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

$$= \sum_{n=0}^{\infty} a_n z^n$$

$$y'(z) = \sum_{n=0}^{\infty} n a_n z^{n-1}$$

$$y''(z) = \sum_{n=0}^{\infty} n(n-1) a_n z^{n-2}$$

plug into DE

$$\sum_{n=0}^{\infty} n(n-1) a_n z^{n-2} + z \cdot \sum_{n=0}^{\infty} n a_n z^{n-1}$$

$$+ 2 \sum_{n=0}^{\infty} a_n z^n = 0$$

$$\sum_{n=0}^{\infty} n(n-1) a_n z^{n-2} - n a_n z^n + 2 a_n z^n = 0$$

$$\sum_{n=0}^{\infty} n(n-1) a_n z^{n-2} + (2-n) a_n z^n = 0$$

$$\sum_{n=2}^{\infty} (n-1)(n)\, a_n z^{n-2} + \sum_{n=0}^{\infty} a_n (2-n) z^n = 0$$

$$m = n-2$$

$$\rightarrow \sum_{m=0}^{\infty} (m+1)(m+2)\, a_{m+2} z^m + \dots$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2)\, a_{n+2} z^n + (2-n) a_n z^n \right] = 0$$

$$(n+1)(n+2)\, a_{n+2} + (2-n)\, a_n = 0$$

$$a_{n+2} = \frac{(n-2)}{(n+1)(n+2)} a_n$$