# Neumann's mutual potential energy of two closed circuits ($s$ and $s'$)

**SOME IMPORTANT TERMS AND DEFINITIONS**

Magnetic shell- A magnetic shell is a thin sheet of magnetic material of uniform thickness magnetized at every point in a direction perpendicular to the surface of the sheet. It may be of any shape (plane, circular or curved). One face of the sheet exhibits north polarity while the other face exhibits south polarity. The magnetic shell may be considered as a polar sheet consisting of a large number of short magnetic dipoles close to each other **with their axis perpendicular to the face of the shell**. If the distribution of the magnetic dipoles over the surface of the shell is uniform, the shell is said to be uniform magnetic shell.

For a magnetic dipole of uniform intensity:

$$\mu = m\tau$$

$$I = \frac{m}{S}$$

$$\phi = I\tau$$

where:
$\mu$ = magnetic moment
$m$ = pole strength of magnetic dipole
$\tau$ = thickness of magnetic dipole
I = intensity of magnetization
$S$ = area of face of magnetic dipole
$\phi$ = strength of magnetic dipole

For a magnetic dipole of a magnetic shell with very small cross sectional area and thickness:

$$I \quad = \frac{\Delta m}{\Delta S}$$

i.e.

$$\Delta m = I \, \Delta S$$

$$\phi \quad = I \, \Delta\tau$$

$$\mu \quad = \Delta m \, \Delta\tau = (I \, \Delta S)\Delta\tau = (I \, \Delta\tau)\Delta S = \phi \, \Delta S$$ **(IMPORTANT)**

## DERIVATION

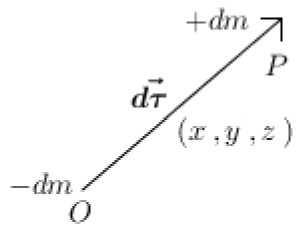
To find potential energy of two closed circuits ($s$ and $s'$), we shall first find potential energy of two magnetic shells of uniform strength and since they are equivalent to two closed circuits having same boundaries carrying constant currents (provided that strength of shell and current of circuit are same), we shall consider potential energy of two magnetic shells of uniform strength as potential energy of two closed circuits having same boundaries carrying constant currents.

To find potential energy of two magnetic shells ($S$ and $S'$) of uniform strength, we shall first find potential energy of two very small elements of shells ($\Delta S$ and $\Delta S'$) and then add them over the two shells.

In this derivation, we have not defined zero potential energy anywhere. So the potential energy formula must contain an arbitrary constant. Let's denote it by $+$ ©.

**(A) Potential energy of $\Delta S$ in a magnetic field $B$ of potential $\Omega$**

Let $\Delta S$ be $OP$ having axis $\overrightarrow{\Delta\tau}$ with $O$ located at $(x, y, z)$.




Let the potential at $P$ be $(\Omega_P + ©)$ and the potential at $O$ be $(\Omega_O + ©)$

Potential energy of $+\Delta m = \Delta m\ (\Omega_P + ©)$
Potential energy of $-\Delta m = -\Delta m\ (\Omega_O + ©)$
**Potential energy of $\Delta S$ in a magnetic field $B$ of potential $\Omega$**
$= \Delta m\ (\Omega_P + ©) - \Delta m\ (\Omega_O + ©)$
$= \Delta m(\Omega_P + © - \Omega_O - ©)$
$= \Delta m\ \Delta\Omega_{(\Delta\tau\ change)} + ©$
where $\Delta\tau$ change is at point $O$

$$= \Delta m\ \Delta\tau\ \frac{\Delta\Omega}{\Delta\tau} + ©$$
$$= \mu \frac{d\Omega}{d\tau} + ©$$
$\{\because \mu = \Delta m\ \Delta\tau\}$
$\{\because \Delta\tau$ is very small, we replaced ratio of very small finite changes with derivative$\}$
$\{\because \Delta\tau$ change is at point $O$, derivative is taken at point $O\}$

$= \phi\left[\vec{\nabla}\Omega.\hat{u}\right]\Delta S + ©$
$\{\because \mu = \phi\ \Delta S\}$
$\{\because$ directional derivative is gradient dotted with unit vector$\}$
where $\hat{u}$ is a unit vector in the direction of $\overrightarrow{\Delta\tau}$

$= \phi\left(-\vec{B}.\overrightarrow{\Delta S}\right) + ©$
$\{\because \vec{B} = -\vec{\nabla}\Omega\}$
where $\overrightarrow{\Delta S}$ is a very small vector in the direction of $\hat{u}$ or $\overrightarrow{\Delta\tau}$

$= -\phi\left(\vec{B}.\overrightarrow{\Delta S}\right) + ©$
$= -\phi\left[\vec{\nabla} \times (\vec{A} + \vec{\nabla}f)\right].\overrightarrow{\Delta S} + ©$
$\{\because \ \vec{\nabla}.\vec{B} = 0\}$
$\{\because$ $\overrightarrow{\Delta S}$ is a very small vector in the direction of $\overrightarrow{\Delta\tau}$, $\overrightarrow{\Delta S}$ can be considered as the normal area vector of the particle $OP$. Also, $\because \Delta S$ is very small, we can consider $\vec{B}$ over it to be uniform. Hence by the definition of curl, we can convert this expression into a line integral.$\}$

$$= -\phi \oint_0^s \left(\vec{A}.\overrightarrow{ds}\right) + ©$$

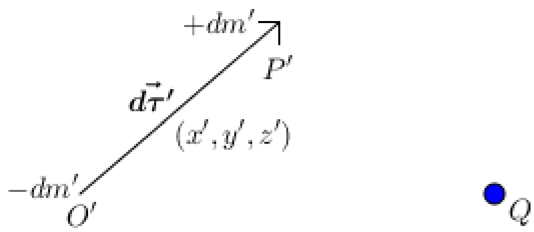
{From the direction of current flowing through the boundary of $\Delta S$, we can find the direction of $\overrightarrow{\Delta\tau}$ or $\overrightarrow{\Delta S}$ by right hand rule. And from it, we can find the direction of line integral, again by right hand rule. Thus we will see that the direction of current flowing through the boundary of $\Delta S$ and the direction of line integral are same. **Thus the direction of line integral is the direction of current flowing through the boundary of $\Delta \boldsymbol{S}$.**}
where $s$ is the boundary of $\Delta S$

If we now suppose that magnetic field $\vec{B}$ and hence $\vec{A}$ are due to another shell $\Delta S'$; we can reduce $\vec{A}$ in terms of $r$, where $r$ is the distance between a point $Q\ (x, y, z)$ and points on the boundary of shell $\Delta S'$

For this, we first need to find:

**(B) Magnetic scalar potential at a point Q $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ due to shell $\Delta \boldsymbol{S'}$ at $(\boldsymbol{x'}, \boldsymbol{y'}, \boldsymbol{z'})$ ……… [$\boldsymbol{\Omega_Q}$]**

Let $\Delta S'$ be $O'P'$ having axis $\overrightarrow{\Delta\tau'}$ with $O'$ located at $(x', y', z')$.

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$\boldsymbol{\Omega_Q}$

$= \Omega_{Q\ (due\ to\ +\Delta m)} + \Omega_{Q\ (due\ to\ -\Delta m)}$

$= k\left(\dfrac{\Delta m'}{P'Q} - \dfrac{\Delta m'}{O'Q}\right) + ©$

where $k$ is Ampere's constant

$= k\ \Delta m'\left(\dfrac{1}{P'Q} - \dfrac{1}{O'Q}\right) + ©$

$= \ k\ \Delta m'\ \Delta\left(\dfrac{1}{r}\right)_{(\Delta\tau'\ change)} + ©$

where $\Delta\tau'$ change is at point $O'$

$= \ k\ \Delta m'\ \Delta\tau'\ \dfrac{\Delta}{\Delta\tau'}\left(\dfrac{1}{r}\right) + ©$

$$= k\,\mu' \frac{d}{d\tau'}\left(\frac{1}{r}\right) + ©$$

$\{\because \mu' = \Delta m'\,\Delta\tau'\}$

$\{\because \Delta\tau'$ is very small, we replaced ratio of very small finite changes with derivative$\}$

$\{\because \Delta\tau'$ change is at point $O$, derivative is taken at point $O\}$

$$= k\,\mu' \left[\vec{\nabla}\left(\frac{1}{r}\right).\hat{u}\right] + ©$$

$\{\because$ directional derivative is gradient dotted with unit vector$\}$

where $\hat{u}$ is a unit vector in the direction of $\overrightarrow{\Delta\tau'}$

$$\hat{u} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k} = u\cos\theta_x\hat{\imath} + u\cos\theta_y\hat{\jmath} + u\cos\theta_z\hat{k} = \frac{dx'}{d\tau'}\hat{\imath} + \frac{dy'}{d\tau'}\hat{\jmath} + \frac{dz'}{d\tau'}\hat{k}$$

$$= l'\hat{\imath} + m'\hat{\jmath} + n'\hat{k}$$

where $l', m', n'$ are the direction cosines of $\hat{u}$ or $\overrightarrow{\Delta\tau'}$.

$$= k\,\mu' \left[\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) l' + \frac{\partial}{\partial y'}\left(\frac{1}{r}\right) m' + \frac{\partial}{\partial z'}\left(\frac{1}{r}\right) n'\right] + © \qquad (26)$$

where $r$ is the distance between point $Q$ $(x, y, z)$ and magnetic particle $O'P'$ at $(x', y', z')$.

Next we need to find:

### (C) $\vec{A}$ at point $Q$ due to a magnetic particle $O'P'$ in terms of $r$

This is done with the help of magnetic scalar potential. We equate the components of "negative gradient of magnetic scalar potential" with the components of "curl of magnetic vector potential".

**Let $F, G, H$ be the components of $\vec{A}$.**

| |
|---|
| By (4): $$\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) = \frac{d}{dr}\left(\frac{1}{r}\right)\frac{\partial r}{\partial x'} = -\frac{1}{r^2}\left(-\frac{\xi}{r}\right) = \frac{\xi}{r^3}$$ and $$\boldsymbol{\frac{\partial}{\partial x}\left(\frac{1}{r}\right)} = \frac{d}{dr}\left(\frac{1}{r}\right)\frac{\partial r}{\partial x} = -\frac{1}{r^2}\frac{\xi}{r} = -\frac{\xi}{r^3} = -\boldsymbol{\frac{\partial}{\partial x'}\left(\frac{1}{r}\right)}$$ Similarly: $$\boldsymbol{\frac{\partial}{\partial y}\left(\frac{1}{r}\right)} = -\boldsymbol{\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)}$$ $$\boldsymbol{\frac{\partial}{\partial z}\left(\frac{1}{r}\right)} = -\boldsymbol{\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)}$$ |
| $$\frac{\partial \mu'}{\partial x} = \frac{\partial \mu'}{\partial y} = \frac{\partial \mu'}{\partial z} = 0$$ (∵ moment of particle $O'P'$ is independent of the position of particle $OP$) |
| $$\frac{\partial l'}{\partial x} = \frac{\partial m'}{\partial x} = \frac{\partial n'}{\partial x} = \frac{\partial l'}{\partial y} = \frac{\partial m'}{\partial y} = \frac{\partial n'}{\partial y} = \frac{\partial l'}{\partial z} = \frac{\partial m'}{\partial z} = \frac{\partial n'}{\partial z} = 0$$ (∵ direction cosine of particle $O'P'$ is independent of the position of particle $OP$) |

$$B_{xQ} = -\frac{\partial \Omega_Q}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left\{k\,\mu'\left[l'\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) + m'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right) + n'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\right] + ©\right\}$$

{by (26)}

$$= k\,\mu'\left[l'\frac{\partial^2}{(\partial x)^2}\left(\frac{1}{r}\right) + m'\frac{\partial^2}{\partial x \partial y}\left(\frac{1}{r}\right) + n'\frac{\partial^2}{\partial x \partial z}\left(\frac{1}{r}\right)\right] \qquad (27©)$$

{since constant +© vanishes in differentiation}

$$= \frac{\partial H_Q}{\partial y} - \frac{\partial G_Q}{\partial z} \qquad (27)$$

$$B_{yQ} = -\frac{\partial \Omega_Q}{\partial y}$$

$$= -\frac{\partial}{\partial y}\left\{k\,\mu'\left[l'\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) + m'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right) + n'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\right] + ©\right\}$$

{by (26)}

$$= k\,\mu'\left[l'\frac{\partial^2}{\partial y\partial x}\left(\frac{1}{r}\right) + m'\frac{\partial^2}{(\partial y)^2}\left(\frac{1}{r}\right) + n'\frac{\partial^2}{\partial y\partial z}\left(\frac{1}{r}\right)\right] \quad (28©)$$

{since constant +© vanishes in differentiation}

$$= \frac{\partial F_Q}{\partial z} - \frac{\partial H_Q}{\partial x} \quad (28)$$

$$B_{zQ} = -\frac{\partial \Omega_Q}{\partial z}$$

$$= -\frac{\partial}{\partial z}\left\{k\,\mu'\left[l'\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) + m'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right) + n'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\right] + ©\right\}$$

{by (26)}

$$= k\,\mu'\left[l'\frac{\partial^2}{\partial z\partial x}\left(\frac{1}{r}\right) + m'\frac{\partial^2}{\partial z\partial y}\left(\frac{1}{r}\right) + n'\frac{\partial^2}{(\partial z)^2}\left(\frac{1}{r}\right)\right] \quad (29©)$$

{since constant +© vanishes in differentiation}

$$= \frac{\partial G_Q}{\partial x} - \frac{\partial F_Q}{\partial y} \quad (29)$$

Solving (27), (28), (29):

$$F_Q = k\,\mu'\left[n'\frac{\partial}{\partial y}\left(\frac{1}{r}\right) - m'\frac{\partial}{\partial z}\left(\frac{1}{r}\right)\right] = k\,\mu'\left[m'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right) - n'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\right]$$

$$G_Q = k\,\mu'\left[l'\frac{\partial}{\partial z}\left(\frac{1}{r}\right) - n'\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\right] = k\,\mu'\left[n'\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) - l'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\right]$$

$$H_Q = k\,\mu'\left[m'\frac{\partial}{\partial x}\left(\frac{1}{r}\right) - l'\frac{\partial}{\partial y}\left(\frac{1}{r}\right)\right] = k\,\mu'\left[l'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right) - m'\frac{\partial}{\partial x'}\left(\frac{1}{r}\right)\right]$$

where $r$ is the distance between point $Q$ $(x, y, z)$ and magnetic particle $O'P'$ at $(x', y', z')$.

Next let's simplify $F_Q$:

$$F_Q = k\,\mu'\left[m'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right) - n'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\right]$$

$$= k\,\phi'\left[m'\frac{\partial}{\partial z'}\left(\frac{1}{r}\right) - n'\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\right]\Delta S'$$

$\{\because \mu' = \phi'\,\Delta S'\}$

$$l' = \frac{dx'}{d\tau'} = \frac{\Delta x'}{\Delta\tau'} = \frac{\Delta S'_x}{\Delta S'} \qquad \Longrightarrow l'\Delta S' = \Delta S'_x$$

{where $\Delta S'_x$ is the projection on $x$- axis of $\overrightarrow{\Delta\tau'}$ when its magnitude is changed to $\Delta S'$}
{i.e. $\overrightarrow{\Delta S'}$ is a very small vector in the direction of $\overrightarrow{\Delta\tau'}$}

Similarly:
$m'\Delta S' = \Delta S'_y$

$n'\Delta S' = \Delta S'_z$

$$= k\,\phi'\left[0 \times \Delta S'_x + \frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\Delta S'_y - \frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\Delta S'_z\right]$$

$$= k\,\phi'\left[0\hat{\boldsymbol{\imath}} + \frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\hat{\boldsymbol{\jmath}} - \frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\hat{\boldsymbol{k}}\right].\left[\Delta S'_x\hat{\boldsymbol{\imath}} + \Delta S'_y\hat{\boldsymbol{\jmath}} + \Delta S'_z\hat{\boldsymbol{k}}\right]$$

$$= k\,\phi'\left(\vec{P}.\overrightarrow{\Delta S'}\right)$$

where $\vec{P} = 0\hat{\boldsymbol{\imath}} + \frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\hat{\boldsymbol{\jmath}} - \frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\hat{\boldsymbol{k}}$

$\{\because$ differentiation is taken at point $(x', y', z')$ on shell $S'$, $P$ is defined at points $(x', y', z')$ on shell $S'\}$

$$\vec{\nabla}.\vec{P} = \frac{\partial P_x}{\partial x'} + \frac{\partial P_y}{\partial y'} + \frac{\partial P_z}{\partial z'}$$

$\{\partial x', \partial y', \partial z'$ because we are taking the divergence of $\vec{P}$ which is defined at points $(x', y', z')$ on shell $S'\}$

$$= \frac{\partial}{\partial x'}[0] + \frac{\partial}{\partial y'}\left[\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\right] - \frac{\partial}{\partial z'}\left[\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\right]$$

$$= 0 + \frac{\partial^2}{\partial y'\partial z'}\left(\frac{1}{r}\right) - \frac{\partial^2}{\partial z'\partial y'}\left(\frac{1}{r}\right)$$

$$= 0$$

$$\therefore P = \vec{\nabla} \times \left(\vec{C} + \vec{\nabla} f\right) = \vec{\nabla} \times \vec{C}$$

where:

$$P_x = (\vec{\nabla} \times \vec{C})_x = \left(\frac{\partial C_z}{\partial y'} - \frac{\partial C_y}{\partial z'}\right)\hat{\boldsymbol{\imath}} = 0\hat{\boldsymbol{\imath}}$$

$$P_y = (\vec{\nabla} \times \vec{C})_y = \left(\frac{\partial C_x}{\partial z'} - \frac{\partial C_z}{\partial x'}\right)\hat{\boldsymbol{\jmath}} = \frac{\partial}{\partial z'}\left(\frac{1}{r}\right)\hat{\boldsymbol{\jmath}}$$

$$P_z = (\vec{\nabla} \times \vec{C})_z = \left(\frac{\partial C_y}{\partial x'} - \frac{\partial C_x}{\partial y'}\right)\hat{\boldsymbol{k}} = -\frac{\partial}{\partial y'}\left(\frac{1}{r}\right)\hat{\boldsymbol{k}}$$

$\{\ \partial x', \partial y', \partial z'$ because $\vec{P}$ is defined at points $(x', y', z')$ on shell $S'\}$

A solution (ignoring $\vec{\nabla} f$ which will cancel out in the forthcoming closed line integral) for the components of $C$ is:

$$C_x = \frac{1}{r};\ C_y = 0;\ C_z = 0$$

$$= k\ \phi' \left[\vec{\nabla} \times (\vec{C} + \vec{\nabla} f)\right] . \overrightarrow{\Delta S'}$$

{$\because \overrightarrow{\Delta S'}$ is a very small vector in the direction of $\overrightarrow{\Delta\tau'}$, $\overrightarrow{\Delta S'}$ can be considered as the normal area vector of the particle $O'P'$. Also, $\because \Delta S'$ is very small, we can consider $\vec{P}$ over it to be uniform. Hence by the definition of curl, we can convert this expression into a line integral.}

$$= k\ \phi' \oint_0^{s'} \vec{C} . \overrightarrow{ds'}$$

{From the direction of current flowing through the boundary of $\Delta S'$, we can find the direction of $\overrightarrow{\Delta\tau'}$ or $\overrightarrow{\Delta S'}$ by right hand rule. And from it, we can find the direction of line integral, again by right hand rule. Thus we will see that the direction of current flowing through the boundary of $\Delta S'$ and the direction of line integral are same. **Thus the direction of line integral is the direction of current flowing through the boundary of $\Delta S'$.**}

where $s'$ is the boundary of $\Delta S'$

$$= k\ \phi' \oint_0^{s'} \left(C_x dx' + C_y dy' + C_z dz'\right)$$

where $\overrightarrow{dx'}, \overrightarrow{dy'}, \overrightarrow{dz'}$ are components of $\overrightarrow{ds'}$

$$= k\ \phi' \oint_0^{s'} \left[\frac{1}{r} dx' + (0 \times dy') + (0 \times dz')\right]$$

{putting the values of $C_x, C_y, C_z$}

$$= k\ \phi' \oint_0^{s'} \frac{dx'}{r}$$

Similarly:

$$G_Q = k\ \phi' \oint_0^{s'} \frac{dy'}{r}$$

$$H_Q = k\ \phi' \oint_0^{s'} \frac{dz'}{r}$$

Therefore:

$$\vec{A} = k\ \phi' \oint_0^{s'} \frac{dx'}{r}(\hat{\imath}) + k\ \phi' \oint_0^{s'} \frac{dy'}{r}(\hat{\jmath}) + k\ \phi' \oint_0^{s'} \frac{dz'}{r}(\hat{k})$$

$$= k\ \phi' \oint_0^{s'} \frac{dx'(\hat{\imath}) + dy'(\hat{\jmath}) + dz'(\hat{k})}{r} = k\ \phi' \oint_0^{s'} \frac{\overrightarrow{ds'}}{r}$$

where $r$ is the distance between point $Q\ (x, y, z)$ and points on the boundary of shell $\Delta S'$

Next we need to find:

**(D) Potential energy of two very small elements of shells ($\Delta S$ and $\Delta S'$)**

By substituting $\vec{A}$ in "potential energy of $\Delta S$ in a magnetic field $B$ of potential $\Omega$" we obtain "potential energy of $\Delta S$ in a magnetic field $B$ due to $\Delta S'$" or "potential energy of $\Delta S$ due to $\Delta S'$" i.e.

$$M = -\phi \oint_0^s (\vec{A}.\overrightarrow{ds}) + ©$$

$$= -\phi \oint_0^s \left( k\,\phi' \oint_0^{s'} \frac{\overrightarrow{ds'}}{r} \right).\overrightarrow{ds} + ©$$

$$= -k\,\phi\phi' \oint_0^s \oint_0^{s'} \frac{\overrightarrow{ds}'.\overrightarrow{ds}}{r} + © \qquad (30©)$$

Here $r$ is the distance between points on the boundaries of shells $\Delta S$ and $\Delta S'$.

**(E) Potential energy of two closed circuits ($s$ and $s'$)**

For this, we need to add potential energies of all $\Delta S$ elements due to all $\Delta S'$ elements. Just like current inside the shells cancel out, the line integrals inside the shells (which are in the direction of current) also cancel out. Thus we are finally left with line integrals in the direction of current flowing through the boundary of shells. Thus **the potential energy of two magnetic shells ($S$ and $S'$) of uniform strength is:**

$$M = -k\ \phi\phi' \oint_0^s \oint_0^{s'} \frac{\overrightarrow{ds}'.\overrightarrow{ds}}{r} + ©$$

where the direction of line integral is the direction of current flowing through the boundary of shells.

i.e. $\overrightarrow{ds}$ **and** $\overrightarrow{ds'}$ **are oriented in the direction of current**.

$$= -k \oint_0^{s'} \oint_0^{s} \frac{\overrightarrow{ds}.\overrightarrow{ds'}}{r} i\, i' + ©$$

{∵ strength of shell is equal to current flowing through its boundary}

Thus **the potential energy of two closed circuits ($s$ and $s'$) is:**

$$M = -k \oint_0^{s'} \oint_0^{s} \frac{\overrightarrow{ds}.\overrightarrow{ds'}}{r} i\, i' + © \tag{30}$$

**It is evident from the equation that when circuits $s$ and $s'$ are interchanged, the potential energy will be same.**