PH-204 Introduction to Physics Simulation

Problem Sheet 3

Handed out 10/11/06

DUE 7/12/06 [FOR ASSESSMENT]

You should hand in your answers to this sheet (on paper!) to Ray Squire.

Q1 The Trapezoidal rule for integration is

$$\int_{x_0}^{x_1} f(x)dx = \frac{1}{2}(f(x_0) + f(x_1))(x_1 - x_0)$$

a) By using the result that this rule is exact for linear functions, give an argument that the error in the above formula can be written as

$$\text{error in Trapezoidal rule} \approx O(\lambda f^{(ii)})$$

where $f^{(ii)}$ is the second derivative of f and λ is some quantity which keeps the dimensions correct. [1 Mark]
b) By dimensional considerations, derive that $\lambda = h^3$. [1 Mark]

Q2 Prove that eq.(1) in the lecture notes (the Lagrangian equation for linear interpolation) is equivalent to eq.(2) (the Taylor series expression). [2 Marks]

Q3 Show that eq.(4) from the lecture notes (the quadratic interpolation formula for equidistant points) is just the usual Taylor expansion:

$$y = y_2 + x\frac{dy}{dx}\,|_0 + \frac{x^2}{2}\frac{d^2y}{dx^2}\,|_0,$$

where $\frac{dy}{dx}|_0$ is the value of $\frac{dy}{dx}$ at $x = 0$. [2 Marks]

Handy Hint: You need to use the finite difference approximation:

$$\frac{dy}{dx}|_0 \approx \frac{1}{2h}(y(+h) - y(-h)).$$

Q4 Obtain an approximate solution to the equation $r(x) = 0$ where $r(x) = e^{-2x} - \frac{1}{2}$ by 2 applications of the Newton-Raphson method, starting at $x = 0$. Compare it with the exact solution. [2 Marks]

Q5 Given the differential equation $\frac{dy}{dx} = x^2 + y$ and the initial conditions $y = 4$ when $x = 1$, find the value of y at $x = 1.2$ using

a) the Euler method with $h = 0.1$ [2 Marks]

b) the Midpoint method with $h = 0.1$ [2 Marks]