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The 86400-second Day

By Frederick T. Collier
Radio Technician
(fred@ftcollier.co.uk)

1. **Introduction**

The units of length and time - the metre and the second – were defined independently of each other in the International System of Units (SI) until 1983, when the metre was re-defined in terms of c, the speed of light, and the second. However, the following observations are matters of fact:

1) that one second is virtually identical to one half the period beat by a simple pendulum of length one metre and;
2) that one second is virtually identical to 30 times the period that light takes to travel the length of the quarter meridian.

These are surprising coincidences that are not explicable by reference to the laws of physics alone.

In a paper by Paolo Agnoli and Giulio D'Agostini, 'Why does the meter beat the second?' (web reference: arxiv:physics/0412078v2, http://www.roma1.infn.it/~dagos/history/sm/), the authors review the steps that led to the choice of the metre as one ten millionth part of the quarter meridian. They discuss the possibility that the length of the seconds pendulum was the starting point in establishing the metre standard. I recommend this work to the reader, and I draw on it in this article.

However, my thesis here is that the choice of the second as 1/86400-th part of the solar day is not itself an arbitrary one, as is generally assumed. I intend to demonstrate how, using astronomical technology no more advanced than that of the early 18^{th} century, it is possible to create a 'natural' period of time based on c and the quarter meridian. I shall show why this natural period must be scaled up by a factor of 30 to make it convenient for people to use. I shall define a procedure for calculating the second that leads to the value 86,400 with an accuracy better than 0.1%. Finally, I shall briefly rehearse the well-known arithmetical arguments that make the 86400-second day a convenient civil standard of time – which also happens to encode astronomical knowledge about the Earth and the heavens.

Before developing the thesis I have proposed, I shall first demonstrate that the expression

$$30 \times \frac{quarter\ meridian}{c}$$

evaluates to a period of time sufficiently close to one second to suggest further investigation is merited. The term 'quarter meridian' I shall take to be 10,002,000 metres to five significant figures. This definition is broad enough to include the distance between the north pole and the equator along any meridian, whilst being precise enough for my purpose. The speed of light, c, is by definition 299,792,458 metres per second exactly. Inserting these values into the expression produces a period of time of 1.0009 seconds, correct to four decimal places. This result is so close to one second that I suggest that the astronomers who chose 86,400 as a suitable divisor of the day for civil use must have been aware of it.

To make the following argument easier to follow, I define here a new term, the c-based second, as a period of time based on the speed of light and the quarter meridian, and specified by the expression given above.

2 **The technology required to establish the c-based second**

The natural philosopher first known to have been in possession of both the astronomical facts and sufficiently precise instruments to calculate the value of the c-based second was the English astronomer James Bradley, Astronomer Royal from 1742. He belonged to a European astronomical tradition that had established an estimate of the size of the Earth's orbit accurate enough to prompt astronomers to attempt to determine distances to the stars by the method of parallax. In the 1720s Bradley tried to fix the distance to the star γ Orionis. Serendipitously, he discovered the effect known as the aberration of light, which he announced to the Royal Society in January 1729.

This is not the place to discuss the aberration of light in detail. Suffice to say that Bradley was able to calculate that c is 10,210 times the mean speed of the Earth in its orbit round the Sun. Today the value is known to be close to 10,066 and this is the figure I shall use in my calculations.

I shall need the value of the light-day later, and here would be an appropriate place to calculate it, taking care to do so without, explicitly or implicitly, introducing the second into the argument. Nasa's Earth Fact Sheet gives a value of 1.4960 x 10^9 m for the semimajor axis of the Earth's orbit round the Sun. To simplify the calculations without sacrificing significant accuracy, I shall take this value to be the radius of the circle approximating the Earth's orbit.

I choose to use 365.25 for the number of solar days in a tropical year as a tribute to Sosigenes of Alexandria, who used this approximation in his design of a new calendar for Julius Caesar in 45 BCE, and so introduced the concept of having a leap year every four years. This is an early example of astronomers simplifying the discoveries they have made in their observatories to make them useful to the public.

Pulling these facts together, we see that the light-day may be calculated thus:-

$$\frac{10066\,x\,2\,x\,\pi\,x\,149.60\,x\,10^{9}\,m}{365.25}$$

giving a value for the light-day of 2.5905 x 10^13 m.

(These days there is a quicker of finding a value for the light-day. Just google it).

3 **A procedure for determining the number of c-based seconds in a day**

Conceptually, determining the number of c-based seconds in a day might involve firing a pulse of light from the equator, reflecting it from the pole, and counting how many times one can repeat this exercise between two successive transits by the Sun of the local meridian. Practically speaking, this idea is a non-starter, and I shall have to calculate the number.

The number of c-based seconds in a solar day is given by the expression:-

$$\frac{one\ solar\ day}{30\,x\,\dfrac{quarter\ meridian}{c}}$$

Multiplying numerator and denominator by c leads to:-

$$\frac{one\ light-day}{30\,x\ quarter\ meridian}$$

and inserting the values already obtained for the light-day and the quarter meridian leads to:-

$$\frac{2.5905\,x\,10^{13}\,m}{30\,x\,1.0002\,x\,10^{7}\,m}$$

which evaluates to 86,333 to the nearest whole number.

4 **The connection with the one-metre pendulum**

Here I should explain why the integer 30 was introduced by the astronomers who devised the 86400-second day. I offer two arguments, the second being a development of the first and superior to it.

The first argument is that just as the quarter meridian is far too long a standard of length to be convenient for people to use, so the ratio of the quarter meridian to c produces a time period which is too short. Thus, the quarter meridian is divided by 10,000,000 to produce a standard length that, for example, a technician can handily manipulate in a workshop; and the ratio of the quarter meridian to c is multiplied by a factor of 30 to produce a period close to that of the human heart beat. A factor of 20 would produce too short a period, and 40 one too long. But 30 is just about right.

However, the typical resting heart rate in adults is 70 beats per minute +/- 15% and I suggest this is too variable a quantity to have been useful in defining the c-based second. Searching for a better alternative, I note that in Appendix A to 'Why does the meter beat the second?' by Agnoli and d'Agostini, it is suggested that the period of a pendulum varies within a range of only +/- 0.15% as the pendulum is moved from a place on the Earth's surface where g has the value 9.80 metres per second squared.

Hence, my second argument, which is to observe first that there must have existed an 'original metre' that preceded the c-based second. Logically, the Earth must have been measured some time before the astronomers were in a position to measure distances to stars by the method of parallax, and discover the aberration of light. Concurrently with the original metre there would have existed a unit of time, which we can posit, without loss of generality, was a decimal-second, similar to the one the French attempted to introduce in 1793. The astronomers would have known the half-period of a one-metre pendulum in terms of the decimal-second. I propose that they chose the integer 30 to specifically fix the c-based second as precisely as possible to the half-period of the one-metre pendulum. That they succeeded is shown by the fact that, expressed in terms of the modern second, the following statements are true:
that the c-based second = 1.0009 seconds;
and that the half-period of a one-metre pendulum = 1.0035 seconds.

Lest the reader imagine that I have pulled the concept of an 'original metre' out of a hat, I refer to Aristotle's work 'On the Heavens' written in the 4th century BCE. Aristotle records that the 'mathematikoi' had calculated the circumference of the Earth to be 400,000 'stadia'. French *savants* of the 18th century believed the Earth had been measured accurately in antiquity, earlier than the Greeks. I am convinced, although I have yet to find the written evidence, that the academicians tasked with devising a new standard of length during the French Revolution deliberately chose to make the metre backward compatible with the measurement reported by Aristotle. Hence, the fact that today we can say that the circumference of the Earth is 40,000 kilometres.

4 **Fitting the c-based second to the 86400-second day**

The astronomers, having determined that there are approximately 86,333 c-based seconds in the day, looked around for a number that would not only be close enough to 86,333 to commemorate what was after all their very significant achievement in discovering the speed of light, but would also be useful for lay purposes. As we know, the number they selected was 86,400.

Put briefly, this number can be factorised as 360 x 240, giving us the circle of 360 degrees. It can also be factorised as 24 x 60 x 60, giving us the day with 24 hours, each hour made of 60 minutes, and each minute of 60 seconds.

5 **Conclusions**

I am well aware that the hypothesis I make in this article, namely that the second is a standard of time derived from a knowledge both of the size of the Earth and the speed of light, is innovative. However, I hope that the facts and observations I have marshalled here are persuasive, although I appreciate that the reasoning will not easily be apprehended by the non-technical reader.

The day of 86,400 seconds is one of our oldest cultural artefacts and it is still in universal use in the modern world. Properly understanding it would tell us much about our prehistory, and I suggest it is time for a concerted, multidisciplinary approach to illuminate the problem. For example, I believe my hypothesis would benefit from Bayesian Analysis, which is a skill that I do not at present possess.

It is unfortunate that I am not able to point to an example of an ancient astronomical telescope for the reader to contemplate. But, the well-known story of the Antikythera Mechanism should teach us that, where future archaeological discoveries are concerned, absolutely nothing should be ruled out definitively.

Fred Collier
www.ftcollier.co.uk

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