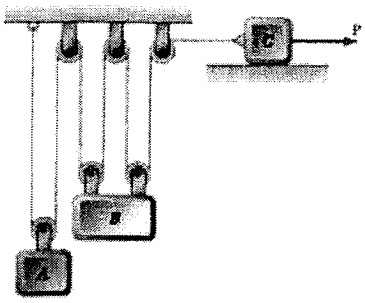
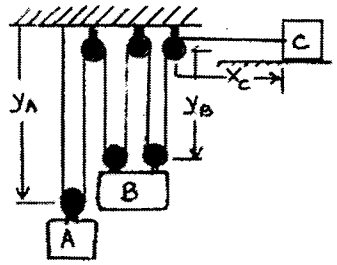


PROBLEM 12.125

The masses of blocks A, B, and C are $m_A = 4$ kg, $m_B = 10$ kg, and $m_C = 2$ kg. Knowing that $P = 0$ and neglecting the masses of the pulleys and the effect of friction, determine (a) the acceleration of each block, (b) the tension in the cord.

SOLUTION



Let the vertical y-coordinates of position of blocks A and B be positive downward and the horizontal x-coordinate of block C be positive to the right as shown.

Constraint of cord. $2y_A + 4y_B + x_C = \text{constant}$

$$2a_A + 4a_B + a_C = 0 \tag{1}$$

Block A: $+\downarrow \Sigma F = ma$: $m_A g - 2T = m_A a_A$

$$a_A = g - \frac{2T}{m_A} \tag{2}$$

Block B: $+\downarrow \Sigma F = ma$: $m_B g - 4T = m_B a_B$

$$a_B = g - \frac{4T}{m_B} \tag{3}$$

Block C: $\pm \Sigma F = ma$: $-T = m_C a_C$

$$a_C = -\frac{T}{m_C} \tag{4}$$

Substituting Eqs. (2), (3) and (4) into Eq. (1),

$$2g - \frac{4T}{m_A} + 4g - \frac{16T}{m_B} - \frac{T}{m_C} = 0$$

$$\left(\frac{4}{m_A} + \frac{16}{m_B} + \frac{1}{m_C}\right)T = 6g$$

$$\left(\frac{4}{2} + \frac{16}{10} + \frac{1}{2}\right)T = (6)(9.81) \qquad \text{or} \qquad T = 18.987 \text{ N}$$

(a) From Eq. (2), $a_A = 9.81 - \dfrac{(2)(18.987)}{4}$ $\quad \mathbf{a}_A = 0.316 \text{ m/s}^2 \downarrow$ ◀

From Eq. (3), $a_B = 9.81 - \dfrac{(4)(18.987)}{10}$ $\quad \mathbf{a}_B = 0.222 \text{ m/s}^2 \downarrow$ ◀

From Eq. (4), $a_C = -\dfrac{18.987}{2}$ $\quad \mathbf{a}_C = 9.49 \text{ m/s}^2 \leftarrow$ ◀

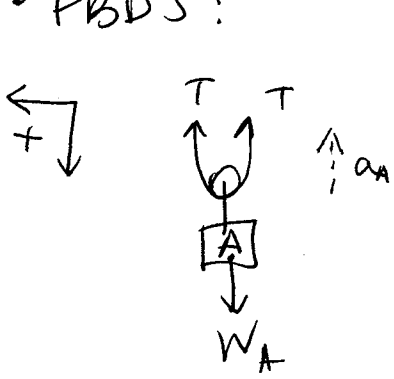
(b) Tension in the cable. $T = 18.99$ N ◀

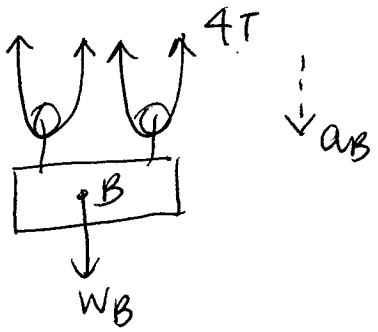
• cable constraint:

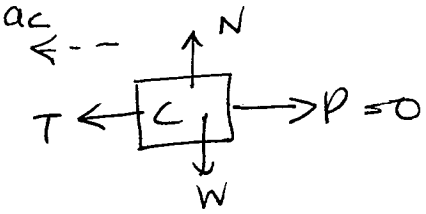
$$2x_A + 4x_B + x_C = \text{const}$$

$$2a_A + 4a_B + a_C = 0 \quad \text{——————} \quad (1).$$

• FBD's:







$$W_A - 2T = m_A a_A \quad (-) \qquad W_B - 4T = m_B a_B \quad (+) \qquad T = m_C a_C \quad (+)$$

$$+a_A = \frac{-W_A + 2T}{m_A} \qquad a_B = \frac{W - 4T}{m_B} \qquad a_C = \frac{T}{m_C}$$

• Plug these 3 equations into (1):

$$2\left(\frac{-W_A + 2T}{m_A}\right) + 4\left(\frac{W_B - 4T}{m_B}\right) + \frac{T}{m_C} = 0. \qquad W = mg$$

Solve for T: $T = 196.2\ N.$