PROBLEM 14.16

A 20-kg projectile is passing through the origin O with a velocity $\mathbf{v}_0 = (60\text{ m/s})\mathbf{i}$ when it explodes into two fragments A and B, of 8-kg and 12-kg, respectively. Knowing that 2 s later the position of fragment A is $(120\text{ m}, -10\text{ m}, -20\text{ m})$, determine the position of fragment B at the same instant. Assume $a_y = -g = -9.81\text{ m/s}^2$ and neglect air resistance.

SOLUTION

Projectile motion = $x = x_0 + V_0 t + \frac{1}{2} g t^2$

The mass center moves as if the projectile had not exploded.

$$\bar{\mathbf{r}} = (\mathbf{v}_0 t) - \left(\frac{1}{2} g t^2\right)\mathbf{j} = (60\mathbf{i})(2) - \left[\frac{1}{2}(9.81)(2)^2\right]\mathbf{j}$$

$$= (120\text{ m})\mathbf{i} - (19.62\text{ m})\mathbf{j}$$

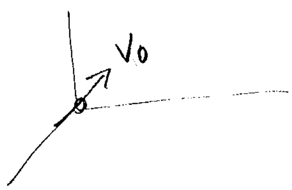
$$(m_A + m_B)\bar{\mathbf{r}} = m_A \mathbf{r}_A + m_B \mathbf{r}_B$$

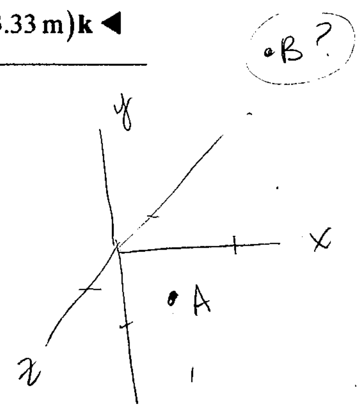
$$\mathbf{r}_B = \frac{1}{m_B}\left[(m_A + m_B)\bar{\mathbf{r}} - m_A \mathbf{r}_A\right]$$

$$= \frac{1}{12}\left[(20)(120\mathbf{i} - 19.62\mathbf{j}) - 8(120\mathbf{i} - 10\mathbf{j} - 20\mathbf{k})\right]$$

$$= 120\mathbf{i} - 26.033\mathbf{j} + 13.333\mathbf{k}$$

$$\mathbf{r}_B = (120.0\text{ m})\mathbf{i} - (26.0\text{ m})\mathbf{j} + (13.33\text{ m})\mathbf{k} \blacktriangleleft$$





SAMPLE PROBLEM 14.1

A 200-kg space vehicle is observed at $t = 0$ to pass through the origin of a newtonian reference frame $Oxyz$ with velocity $\mathbf{v}_0 = (150\text{ m/s})\mathbf{i}$ relative to the frame. Following the detonation of explosive charges, the vehicle separates into three parts A, B, and C, of mass 100 kg, 60 kg, and 40 kg, respectively. Knowing that at $t = 2.5$ s the positions of parts A and B are observed to be $A(555, -180, 240)$ and $B(255, 0, -120)$, where the coordinates are expressed in meters, determine the position of part C at that time.

SOLUTION

$V = \frac{\Delta r}{\Delta t}$

$V = \text{const}$

$X = X_0 + vt$

Since there is no external force, the mass center G of the system moves with the constant velocity $\mathbf{v}_0 = (150\text{ m/s})\mathbf{i}$. At $t = 2.5$ s, its position is

$$\bar{\mathbf{r}} = \mathbf{v}_0 t = (150\text{ m/s})\mathbf{i}(2.5\text{ s}) = (375\text{ m})\mathbf{i}$$

def $\bar{r} = \frac{\Sigma m_i r_i}{\Sigma m_i}$

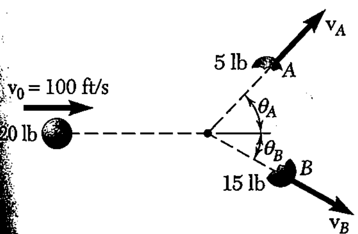
Recalling Eq. (14.12), we write

$$m\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$$

$$(200\text{ kg})(375\text{ m})\mathbf{i} = (100\text{ kg})[(555\text{ m})\mathbf{i} - (180\text{ m})\mathbf{j} + (240\text{ m})\mathbf{k}] + (60\text{ kg})[(255\text{ m})\mathbf{i} - (120\text{ m})\mathbf{k}] + (40\text{ kg})\mathbf{r}_C$$

$$\mathbf{r}_C = (105\text{ m})\mathbf{i} + (450\text{ m})\mathbf{j} - (420\text{ m})\mathbf{k} \quad ◀$$

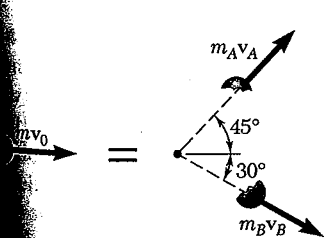
SAMPLE PROBLEM 14.2



A 20-lb projectile is moving with a velocity of 100 ft/s when it explodes into two fragments A and B, weighing 5 lb and 15 lb, respectively. Knowing that immediately after the explosion, fragments A and B travel in directions defined respectively by $\theta_A = 45°$ and $\theta_B = 30°$, determine the velocity of each fragment.

SOLUTION

Since there is no external force, the linear momentum of the system is conserved, and we write



$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m\mathbf{v}_0$$

$$(5/g)\mathbf{v}_A + (15/g)\mathbf{v}_B = (20/g)\mathbf{v}_0$$

$\xrightarrow{+}$ x components: $\quad 5v_A \cos 45° + 15v_B \cos 30° = 20(100)$

$+\uparrow$ y components: $\quad 5v_A \sin 45° - 15v_B \sin 30° = 0$

Solving simultaneously the two equations for v_A and v_B, we have

$$v_A = 207\text{ ft/s} \qquad v_B = 97.6\text{ ft/s}$$

$$\mathbf{v}_A = 207\text{ ft/s} \angle 45° \qquad \mathbf{v}_B = 97.6\text{ ft/s} \angle 30° \quad ◀$$