One mole of a gas with $\gamma = 4/3$ goes over the cycle ABCA as in Figure 2 where one of AB or AC is isothermal and the other adiabatic. (You figure out which.) Write down the (P, V, T) coordinates of A, B and C (some of which are already given). What is the work done in each part of the cycle and the heat absorbed or rejected in the full cycle?

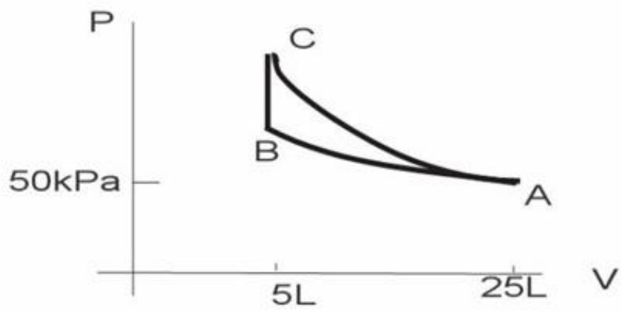

FIG. 2. The gas goes in a loop ABCA, where either AB or AC is isothermal and the other is adiabatic.

Let's first figure out which of AB and AC is the isothermal and which is the adiabatic step. Along the isothermal step we know that:

$$P_A V_A = P_{isothermal} V_{isothermal}$$

But since the pressure and volume is known for point A, and the volumes of both points B and C are know and the same:

$$P_{isothermal} = \frac{P_A V_A}{V_B} = \frac{(50kPa)(25L)}{5L} = 250kPa$$

Along the adiabatic step we know that:

$$P_A V_A^{\gamma} = P_{adiabatic} V_{adiabatic}^{\gamma}$$

which can be solved for the pressure resulting from the adiabatic step:

$$P_{adiabatic} = P_A \left(\frac{V_A}{V_{adiabatic}} \right)^{\gamma} = (50kPa) \left(\frac{25L}{5L} \right)^{4/3} \approx 427.5kPa$$

Since we can see from the diagram that point C has a higher pressure than point B we know that AC must be the adiabatic process and AB must be the isothermal process.

Let's now track the work done in each part of the cycle. Along the isothermal path from A to B:

$$W_{A \to B} = -P_A V_A \ln\left(\frac{V_A}{V_B} \right) = -(50kPa)(25L)(10^{-3}\, m^3/L) \ln\left(\frac{25L}{5L} \right) \approx -2012J$$

The minus sign here indicates that work is done on the gas in going from point A to point B. The step that goes from B to C is at constant volume and so does no work.

$$W_{B \to C} = 0J$$

The step that goes from C to A is adiabatic and does work:

$$W_{C\to A} = \frac{P_C V_C - P_A V_A}{\gamma - 1} = \frac{(427.5kPa)(5L)(10^{-3}\,m^3/L) - (50kPa)(25L)(10^{-3}\,m^3/L)}{4/3 - 1} \approx 2662J$$

So the total work is:

$$W_{ABCA} = W_{A\to B} + W_{B\to C} + W_{C\to A} \approx -2012J + 0J + 2662J = 650J$$

Since the system returns to its original state, its internal energy doesn't have a net change. This means that any work done by the gas must have been absorbed as heat from its surroundings. So the heat absorbed in the full cycle is about 650 *J*.

To complete the (P,V,T) coordinates for the three points, we can make use of the fact that $T_A = T_B$ since they are connected by an isothermal process. To get values for T_A and T_C we just make use of the ideal gas law:

$$T = \frac{PV}{nR}$$

So, using the previously found values:

$$T_A = \frac{(50kPa)(25L)(10^{-3}\,m^3/L)}{(1mol)\left(8.314\frac{J}{mol\cdot K}\right)} \approx 150K$$

$$T_C = \frac{(427.5kPa)(5L)(10^{-3}\,m^3/L)}{(1mol)\left(8.314\frac{J}{mol\cdot K}\right)} \approx 257K$$

Which finally lets us write the final (P,V,T) coordinates for the three points.

Point A: (50 *kPa*, 25 *L*, 150 *K*)
Point B: (250 *kPa*, 5 *L*, 150 *K*)
Point C: (427.5 *kPa*, 5 *L*, 257 *K*)