I have the equation $\nabla^2\varphi = 0$ and want to satisfy $\varphi = -1$ on $r = 2$ and $\varphi = -1/4$ on $r = 1$ (the inner boundary of the annulus). The annulus has inner radius 1 and outer radius 2.

I now have 2 fundamental solutions and by superpostion the sum of fundamental solutions is also a solution. Hence have,

$$\varphi = \sum_{j=1}^{N} c_j\, log\, r_j + \sum_{j=1}^{M} d_j\, log\, r_j^{in} \tag{1}$$

Where N is the number of singularities on the ring of sigularities on the outside the domain and M is the number of singularities on the ring of singularities on the inside of the domain. And $r_j^{in} = \sqrt{[(x-\xi_j^{in})^2 + (y-\eta_j^{in})^2]}$ where $(\xi_j^{in}, \eta_j^{in})$ is the co-ord of the jth singularity on the inside ring. So c_j and d_j are to be found. I want to satisfy the boundary conditions so want to minimise the following

$$I = \underbrace{\int_{\Gamma} |\varphi + 1|^2 \mathrm{d}s}_{1} + \underbrace{\int_{\Gamma_{in}} |\varphi + 1/4|^2 \mathrm{d}s}_{2} \tag{2}$$

Where Γ is the outer boudary and Γ_{in} is the inner boundary. I is now a function of c_j and d_j. Continue to calculate the partial differential of I with respect to c_j and d_j separately and put them equal to 0. I have done this and subsequesntly built a $2N$x$2N$ matrix multiplying $[c_j, d_j]$.