3.2.14 $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b}\,1_2 + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$

Left side: $\vec{\sigma} \equiv \hat{x}\,\sigma_1 + \hat{y}\,\sigma_2 + \hat{z}\,\sigma_3 = \langle \sigma_1, \sigma_2, \sigma_3 \rangle$

$$\vec{\sigma}\cdot\vec{a} = a_x\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + a_y\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + a_z\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$= \begin{pmatrix}0 & a_x\\ a_x & 0\end{pmatrix} + \begin{pmatrix}0 & -a_y i\\ a_y i & 0\end{pmatrix} + \begin{pmatrix}a_z & 0\\ 0 & -a_z\end{pmatrix}$$

$$\vec{\sigma}\cdot\vec{b} = \begin{pmatrix}0 & b_x\\ b_x & 0\end{pmatrix} + \begin{pmatrix}0 & -b_y i\\ b_y i & 0\end{pmatrix} + \begin{pmatrix}b_z & 0\\ 0 & -b_z\end{pmatrix}$$

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \begin{pmatrix}0 & a_xb_x\\ a_xb_x & 0\end{pmatrix} + \begin{pmatrix}0 & -a_yb_y\\ -a_yb_y & 0\end{pmatrix} + \begin{pmatrix}a_zb_z & 0\\ 0 & a_zb_z\end{pmatrix}$$

Right side: 1.) $\vec{a}\cdot\vec{b}\,1_2 = (a_xb_x + a_yb_y + a_zb_z)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$= \begin{pmatrix}a_xb_x & 0\\ 0 & a_xb_x\end{pmatrix} + \begin{pmatrix}a_yb_y & 0\\ 0 & a_yb_y\end{pmatrix} + \begin{pmatrix}a_zb_z & 0\\ 0 & a_zb_z\end{pmatrix}$$

2.) $i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$ a.) $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = \hat{x}(a_yb_z - a_zb_y) + \hat{y}(a_zb_x - a_xb_z) + \hat{z}(a_xb_y - a_yb_x)$

$$= \langle a_yb_z - a_zb_y,\ a_zb_x - a_xb_z,\ a_xb_y - a_yb_x \rangle$$

b.) $i\vec{\sigma} = \left\langle i\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \right\rangle$

$$= \left\langle \begin{pmatrix}0 & i\\ i & 0\end{pmatrix}, \begin{pmatrix}0 & -i^2\\ i^2 & 0\end{pmatrix}, \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} \right\rangle$$