Azimuthal B-Field Of Loose Coil

I. Cartesian Coordinates:

II. Cylindrical Coordinates:

A. Define Parameters:

- h = pitch of coil

- R = radius of coil

- $r = R$, $\phi = (\frac{2\pi}{h})z'$, $z' \in [-\infty,+\infty]$ describes the current on the coil

- $dl = (dr)\,\hat{r} + (Rd\phi)\,\hat{\phi} + (dz')\,\hat{z} = \left(\left(\frac{2\pi R}{h}\right)dz'\right)\hat{\phi} + (dz')\,\hat{z}$

- $\vec{r} = (R)\,\hat{r} + ((\frac{2\pi}{h})z')\,\hat{\phi} + (z'-z)\,\hat{z}$

- $\|\vec{r}\| = \left[(z' - z)^2 + r^2 + R^2 - 2\,Rr\mathrm{Cos}(\phi - \phi')\right]^{1/2} = \left[(z' - z)^2 + R^2\right]^{1/2}$

B. Integrals And Limits Needed For Biot Savart Integrals:

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Integrate[1 / (r^2 + x^2)^(3 / 2), x]
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$$\frac{x}{r^2\sqrt{r^2 + x^2}}$$

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Integrate[x / (r^2 + x^2)^(3 / 2), x]
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$$-\frac{1}{\sqrt{r^2 + x^2}}$$

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Limit[1 / Sqrt[a^2 + (-b + x)^2], x → Infinity]
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0

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Limit[x / Sqrt[a^2 + (-b + x)^2], x → Infinity]
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1

C. Compute Biot Savart Integrals:

- $B(\vec{r}) = \frac{\mu I}{4\pi} \int \frac{dl \, x \, \vec{r}}{||\vec{r}\,||^3}$

- $dl \times \vec{r} = [((\frac{2\pi R}{h})dz')(z'-z) - \left(\frac{2\pi z'}{h}\right)(dz')\Big] \hat{r} - [(0) - (R)(dz')]\, \hat{\phi} + [(0)-((\frac{2\pi R}{h})dz')(R)]\, \hat{z}$

$= [((\frac{2\pi R}{h})dz')(z'-z) - \left(\frac{2\pi z'}{h}\right)(dz')\Big] \hat{r} + [(R)(dz')]\, \hat{\phi} - [(\frac{2\pi R^2}{h})dz']\, \hat{z}$

$$B_r(z) = \frac{\mu I}{4\pi} \int \frac{\left[\left(\left(\frac{2\pi R}{h}\right)dz'\right)(z'-z) - \left(\frac{2\pi z'}{h}\right)(dz')\right]\hat{r}}{\left[(z'-z)^2 + R^2\right]^{3/2}}$$

$$= \frac{\mu I}{4\pi}\left(\int \frac{\left[\left(\left(\frac{2\pi R}{h}\right)dz'\right)(z'-z)\right.}{\left[(z'-z)^2 + R^2\right]^{3/2}} - \int \frac{\left.\left(\frac{2\pi z'}{h}\right)(dz')\right]}{\left[(z'-z)^2 + R^2\right]^{3/2}}\right)\hat{r}$$

$$= \frac{\mu I}{4\pi}\left(\left(\frac{2\pi}{Rh}\right)\frac{(-1)}{\left[(z'-z)^2 + R^2\right]^{1/2}}\,\Big|^{\infty} - \left(\frac{2\pi}{h}\right)\left[\frac{-1}{\left[(z'-z)^2 + R^2\right]^{1/2}}\,\Big|^{\infty} + \frac{z(z'-z)}{(R^2)\left[(z'-z)^2 + R^2\right]^{1/2}}\,\Big|^{\infty}\right]\right)\hat{r}$$

$$= \frac{\mu I}{4\pi}\left(\left(\frac{2\pi}{Rh}\right)[0] - \left(\frac{2\pi}{h}\right)\left[(0) + \left(\frac{z}{R^2}\right)[(1) - (-1)]\right]\right)\hat{r}$$

$$= \frac{\mu I}{4\pi}\left(\frac{2\pi}{h}\right)\left[\left(\frac{z}{R^2}\right)[2]\right]\hat{r}$$

$$= \frac{\mu I z}{h R^2}\,\hat{r}$$

$$B_\phi(z) = \frac{\mu I}{4\pi}\int \frac{(R)\,dz'}{\left[(z'-z)^2 + R^2\right]^{3/2}}\,\hat{\phi}$$

$$= \frac{\mu I R}{4\pi}\left[\frac{(z'-z)}{(R^2)\left[(z'-z)^2 + R^2\right]^{1/2}}\right]\Big|^{\infty}\,\hat{\phi}$$

$$= \frac{\mu I}{4\pi R}\,[(1 - (-1)) - (0)]\,\hat{\phi}$$

$$= \frac{\mu I}{2\pi R}\,\hat{\phi}$$

$$B_z(z) = -\frac{\mu I}{4\pi}\int\frac{\left(\frac{2\pi R^2}{h}\right)dz'}{\left[(z'-z)^2+R^2\right]^{3/2}}\,\hat{z}$$

$$= -\frac{\mu I R^2}{2h}\left[\frac{(z'-z)}{(R^2)\left[(z'-z)^2+R^2\right]^{1/2}}\right]\Big|^{\infty}\hat{z}$$

$$= -\frac{\mu I R^2}{2h}\left[(1-(-1))-(0)\right]\hat{z}$$

$$= -\frac{\mu I R^2}{h}\hat{z}$$

$$\vec{B}(r,\ \phi,\ z) = \left(\frac{\mu I z}{hR^2}\right)\hat{r} + \left(\frac{\mu I}{2\pi R}\right)\hat{\phi} + \left(-\frac{\mu I R^2}{h}\right)\hat{z}$$

D. Taking Limiting Case Of h → ∞:

- If h (the pitch of the coil) approaches infinity, then the coil will be the equivalent of a straight line current with the appropriate magnetic field:

- $\lim_{h\to\infty}\vec{B}(r,\ \phi,\ z) = \lim_{h\to\infty}\left(\frac{\mu I z}{hR^2}\right)\hat{r} + \left(\frac{\mu I}{2\pi R}\right)\hat{\phi} + \left(-\frac{\mu I R^2}{h}\right)\hat{z} = (0)\,\hat{r} + \left(\frac{\mu I}{2\pi R}\right)\hat{\phi} + (0)\,\hat{z} = \left(\frac{\mu I}{2\pi R}\right)\hat{\phi}$

- This is the correct field with the correct direction, $\hat{\phi}$ is perpendicular to the radius vector by definition.

E. Finding the Phase Between The Helix Coil And The Magnetic Field:

- Curl Of Helix:

- $\nabla \times \left[(R)\,\hat{r} + \left(\frac{2\pi}{h}z'\right)\hat{\phi} + (z')\,\hat{z}\right] = \left(-\frac{2\pi}{h}\right)\hat{r} + (0)\,\hat{\phi} + (0)\,\hat{z} = \left(-\frac{2\pi}{h}\right)\hat{r}$