Unit 1
Introduction

INFINITE SERIES :

An expression of the form $u_1 + u_2 + u_3 + \ldots\ldots + u_n + \ldots\ldots$ in which the terms $u_1, u_2, \ldots\ldots, u_n, \ldots.$ occur according to some definite law is called a **series.**

A series which is having finite number of terms is called a **finite series** whereas a series which is having an infinite number of terms is called an **infinite series.**

The infinite series is usually denoted by $u_1 + u_2 + u_3 + \ldots\ldots + u_n + \ldots\ldots$ or by $\sum u_n$.

CONVERGENCE, DIVERGENCE AND OSCILLATION OF A SERIES :

Consider the infinite series $\sum u_n = u_1 + u_2 + u_3 + \ldots\ldots + u_n + \ldots\ldots \infty$

and let the sum of the first n terms be $s_n = u_1 + u_2 + u_3 + \ldots\ldots + u_n$

Now, we can see that s_n is a function of n and as n increases indefinitely three possibilities arises :

(i) **If s_n tends to a finite limit as $n \to \infty$** then the series $\sum u_n$ is said to be **convergent.**

(ii) If s_n **tends to a $\pm\infty$ as $n \to \infty$** then the series $\sum u_n$ is said to be **divergent.**

(iii) If s_n **does not tend to a unique limit as $n \to \infty$** then the series $\sum u_n$ is said to be **oscillatory.**

Ex. 1 Examine for convergence the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$

Solution :

Step I : Finding s_n :

From the given series, we get expression of s_n as :

$$\therefore \quad s_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n}$$

$$\therefore \quad s_n = \frac{1}{2} \cdot \frac{1 - \left(\frac{1}{2}\right)^n}{1 - \left(\frac{1}{2}\right)}$$