given X, Y random variables

with joint density probability:

$$f_{X,Y}(x,y) = C \cdot \frac{x+y}{(x^2+y^2)^2} \mathbb{1}(0 < \sqrt{2}y < x, \; 1 < x^2+y^2)$$

$f_{X,Y}(x,y) > 0 \Rightarrow C > 0$

$x, y > 0$

(a) find C

$$\iint f_{X,Y}(x,y)\,dx\,dy = 1$$

Polar cor.

$x = r\cos\theta$

$y = r\sin\theta$

$$\begin{cases} 1. \; 1 < r^2 \\ 2. \; 0 < \theta < tg^{-1}\left(\frac{1}{\sqrt{2}}\right) \end{cases}, \quad dx\,dy = r\,dr\,d\theta$$

$$C \cdot \int_0^{tg^{-1}\left(\frac{1}{\sqrt{2}}\right)} \int_1^{\infty} \frac{r\cos(\theta) + r\sin\theta}{(r^2\cos^2\theta + r^2\sin^2\theta)^2} \, r\,dr\,d\theta = 1$$

$$C \cdot \int_0^{tg^{-1}\left(\frac{1}{\sqrt{2}}\right)} (\cos(\theta) + \sin(\theta))\,d\theta \int_1^{\infty} \frac{1}{r^2}\,dr = 1$$

$C \cdot 0.76 \cdot 1 = 1$

$C = 1.315$

(b) find the expected value of X

marginal dist. of X :

$$g(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$\begin{cases} 0 < y < \frac{x}{\sqrt{2}} \\ 1 - x^2 < y^2 \quad \Rightarrow \quad \sqrt{1-x^2} < y \end{cases} \Rightarrow \sqrt{1-x^2} < y < \frac{x}{\sqrt{2}}$$

$$g(x) = c \cdot \int_{\sqrt{1-x^2}}^{\frac{x}{\sqrt{2}}} \frac{x+y}{(x^2+y^2)^2}\,dy$$

$$= c \cdot \left[\underset{(1)}{\int_{\sqrt{1-x^2}}^{x/\sqrt{2}} \frac{x}{(x^2+y^2)^2}\,dy} + \underset{(2)}{\int_{\sqrt{1-x^2}}^{x/\sqrt{2}} \frac{y}{(x^2+y^2)^2}\,dy} \right]$$

$$* \left[\begin{array}{l} (1) \int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\, tg^{-1}\frac{x}{a} \\ (2) \int \frac{x\,dx}{(x^2+a^2)^2} = -\frac{1}{2(x^2+a^2)} \end{array} \right]$$

$$= c \cdot \left[\frac{y}{2x(y^2+x^2)} + \frac{1}{2x^2}\, tg^{-1}\left(\frac{y}{x}\right) \Big|_{\sqrt{1-x^2}}^{\frac{x}{\sqrt{2}}} \right.$$

$$- \frac{1}{2(y^2+x^2)} \Big|_{\sqrt{1-x^2}}^{\frac{x}{\sqrt{2}}} \longrightarrow ?$$