

Figure 1: Problem diagram
Start with Eq. (1), the Biot-Savart Law:

$$
\begin{equation*}
d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{i d \vec{s} \times \hat{r}}{r^{2}} \tag{1}
\end{equation*}
$$

From the diagram:

$$
\begin{array}{r}
r=\sqrt{x^{2}+y^{2}} \\
x=r \sin (\theta) \\
y=r \cos (\theta) \\
d \vec{s}=d \vec{y}
\end{array}
$$

Substitute into Eq. (1):

$$
\begin{equation*}
d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{i d \vec{y} \times \hat{r}}{x^{2}+y^{2}} \tag{2}
\end{equation*}
$$

Taking the cross product, $i d \vec{y} \times \hat{r}=d y \sin (\theta)(-\hat{k})$. Substitute in:

$$
\begin{equation*}
d \vec{B}=\frac{\mu_{0} i}{4 \pi} \frac{d y \sin (\theta)}{x^{2}+y^{2}}(-\hat{k}) \tag{3}
\end{equation*}
$$

From above, $\sin (\theta)=\frac{x}{r}$ :

$$
\begin{equation*}
d \vec{B}=\frac{\mu_{0} i}{4 \pi} \frac{x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}(-\hat{k}) \tag{4}
\end{equation*}
$$

Find $\vec{B}$ by taking the integral over the infinite line:

$$
\begin{equation*}
\vec{B}=\frac{\mu_{0} i}{4 \pi} \int_{-\infty}^{\infty} \frac{x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}(-\hat{k}) \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\vec{B}=2 \frac{\mu_{0} i}{4 \pi} \int_{0}^{\infty} \frac{x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}(-\hat{k}) \tag{6}
\end{equation*}
$$

Integrate (I used Wolfram|Alpha) to get:

$$
\begin{equation*}
\vec{B}=2 \frac{\mu_{0} i}{4 \pi}\left[\frac{y}{x \sqrt{x^{2}+y^{2}}}\right]_{0}^{\infty}(-\hat{k}) \tag{7}
\end{equation*}
$$

The indefinite integral evaluates to:

$$
\begin{equation*}
\vec{B}=\frac{\mu_{0} i}{2 \pi x}(-\hat{k}) \tag{8}
\end{equation*}
$$

We can confirm this using Ampere's Law (left as an exercise to the reader...)

