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Born rule

The **Born rule** (also called the **Born law**, **Born's postulate**, **Born's rule**, or **Born's law**) is a key postulate of quantum mechanics which gives the probability that a measurement of a quantum system will yield a given result.[1] In its simplest form, it states that the probability density of finding a particle at a given point is proportional to the square of the magnitude of the particle's wavefunction at that point. It was formulated by German physicist Max Born in 1926.

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Details

The Born rule states that if an observable corresponding to a self-adjoint operator A with discrete spectrum is measured in a system with normalized wave function $|\psi\rangle$ (*see* Bra–ket notation), then

- the measured result will be one of the eigenvalues λ of A, and
- the probability of measuring a given eigenvalue λ_i will equal $\langle\psi|P_i|\psi\rangle$, where P_i is the projection onto the eigenspace of A corresponding to λ_i.

  (In the case where the eigenspace of A corresponding to λ_i is one-dimensional and spanned by the normalized eigenvector $|\lambda_i\rangle$, P_i is equal to $|\lambda_i\rangle\langle\lambda_i|$, so the probability $\langle\psi|P_i|\psi\rangle$ is equal to $\langle\psi|\lambda_i\rangle\langle\lambda_i|\psi\rangle$. Since the complex number $\langle\lambda_i|\psi\rangle$ is known as the *probability amplitude* that the state vector $|\psi\rangle$ assigns to the eigenvector $|\lambda_i\rangle$, it is common to describe the Born rule as saying that probability is equal to the amplitude-squared (really the amplitude times its own complex conjugate). Equivalently, the probability can be written as $|\langle\lambda_i|\psi\rangle|^2$.)

In the case where the spectrum of A is not wholly discrete, the spectral theorem proves the existence of a certain projection-valued measure Q, the spectral measure of A. In this case,

- the probability that the result of the measurement lies in a measurable set M is given by $\langle\psi|Q(M)|\psi\rangle$.

Given a wave function ψ for a single structureless particle in position space, implies that the probability density function $p(x,y,z)$ for a measurement of the position at time t_0 is

$$p(x,y,z) = |\psi(x,y,z,t_0)|^2.$$

History

The Born rule was formulated by Born in a 1926 paper.[2] In this paper, Born solves the Schrödinger equation for a scattering problem and, inspired by Einstein's work on the photoelectric effect,[3] concludes, in a footnote, that the Born rule gives the only possible interpretation of the solution. In 1954, together with Walther Bothe, Born was awarded the Nobel Prize in Physics for this and other work.[3] John von Neumann discussed the application of spectral theory to Born's rule in his 1932 book.[4]

Interpretations

Within the Quantum Bayesianism interpretation of quantum theory, the Born rule is seen as an extension of the standard Law of Total Probability, which takes into account the Hilbert space dimension of the physical system involved.[5] In the ambit of the so-called Hidden-Measurements Interpretation of quantum mechanics the Born rule can be derived by averaging over all possible measurement-interactions that can take place between the quantum entity and the measuring system.[6][7] It has been claimed that Pilot wave theory can also statistically derive Born's law.[8] While it has been claimed that Born's law can be derived from the many-worlds interpretation, the existing proofs have been criticized as circular.[9] Kastner claims that the transactional interpretation is unique in giving a physical explanation for the Born rule.[10]

See also

- Einstein and the quantum
- Gleason's theorem
- Unitarity

References

1. The time evolution of a quantum system is entirely deterministic according to the Schrödinger equation. It is through the Born Rule that probability enters into the theory.
2. Born, Max (1926). "I.2". In Wheeler, J. A.; Zurek, W. H. (eds.). *Zur Quantenmechanik der Stoßvorgänge* [*On the quantum mechanics of collisions*]. *Zeitschrift für Physik*. **37**. Princeton University Press (published 1983). pp. 863–867. Bibcode:1926ZPhy...37..863B (https://ui.adsabs.harvard.edu/abs/1926ZPhy...37..863B). doi:10.1007/BF01397477 (https://doi.org/10.1007%2FBF01397477). ISBN 978-0-691-08316-2.
3. Born, Max (11 December 1954). "The statistical interpretation of quantum mechanics" (https://www.nobelprize.org/uploads/2018/06/born-lecture.pdf) (PDF). *www.nobelprize.org*. nobelprize.org. Retrieved 7 November 2018. "Again an idea of Einstein's gave me the lead. He had tried to make the duality of particles - light quanta or photons - and waves comprehensible by interpreting the square of the optical wave amplitudes as probability density for the occurrence of photons. This concept could at once be carried over to the psi-function: $|psi|^2$ ought to represent the probability density for electrons (or other particles)."
4. Neumann (von), John (1932). *Mathematische Grundlagen der Quantenmechanik* [*Mathematical Foundations of Quantum Mechanics*]. Translated by Beyer, Robert T. Princeton University Press (published 1996). ISBN 978-0691028934.
5. Christopher A. Fuchs (2010). "QBism, the Perimeter of Quantum Bayesianism". arXiv:1003.5209 (https://arxiv.org/abs/1003.5209) [quant-ph (https://arxiv.org/archive/quant-ph)].
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8. Towler, Mike. "Pilot wave theory, Bohmian metaphysics, and the foundations of quantum mechanics" (https://www.tcm.phy.cam.ac.uk/~mdt26/PWT/lectures/bohm7.pdf) (PDF).
9. Landsman, N. P. (2008). "The conclusion seems to be that no generally accepted derivation of the Born rule has been given to date, but this does not imply that such a derivation is impossible in principle" (https://www.math.ru.nl/~landsman/Born.pdf) (PDF). In Weinert, F.; Hentschel, K.; Greenberger, D.; Falkenburg, B. (eds.). *Compendium of Quantum Physics*. Springer. ISBN 3-540-70622-4.
10. Kastner, R. E. (2013). *The Transactional Interpretation of Quantum Mechanics* (https://archive.org/details/transactionalint00kast). Cambridge University Press. p. 35 (https://archive.org/details/transactionalint00kast/page/n44). ISBN 978-0-521-76415-5.

External links

- Quantum Mechanics Not in Jeopardy: Physicists Confirm a Decades-Old Key Principle Experimentally (https://www.sciencedaily.com/releases/2010/07/100722142640.htm) ScienceDaily (July 23, 2010)