

$f(z)$ is analytical in D where D is a connected and open domain in \mathbb{C} ,
let $D^* = \{z : \bar{z} \in D\}$. Define $g(z) = \overline{f(\bar{z})}$, show that $g(z)$ is analytical in D^* .
work done so far:

if $f(z)$ is analytical in D : satisfy CR-eq.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\overline{f(\bar{z})} = u(\bar{z}) - iv(\bar{z})$$

I want to show that:

$$\frac{\partial u(\bar{z})}{\partial x} = -\frac{\partial v(\bar{z})}{\partial y}$$

and:

$$\frac{\partial u(\bar{z})}{\partial y} = \frac{\partial v(\bar{z})}{\partial x}$$

reverse sign, because i have:

$$\frac{\partial u}{\partial x} = \frac{\partial(-v)}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial(-v)}{\partial x}$$