



Figure 1: Differential fluid element with x-direction stresses

Force Balance:

$$\sum f = m \frac{Du}{Dt} \quad (1)$$

Summing up the force vectors in the x direction,

$$\sum f = (\sigma_1 + \sigma_2)\Delta y\Delta z + (\sigma_3 + \sigma_4)\Delta x\Delta z + (\sigma_5 + \sigma_6)\Delta x\Delta y + mg \quad (2)$$

Substituting (2) into (1) and dividing by $\rho\Delta x\Delta y\Delta z$,

$$\frac{Du}{Dt} = \frac{1}{\rho} \left[\frac{\sigma_1 + \sigma_2}{\Delta x} + \frac{\sigma_3 + \sigma_4}{\Delta y} + \frac{\sigma_5 + \sigma_6}{\Delta z} \right] + g \quad (3)$$

I know the result is supposed to be

$$\frac{Du}{Dt} = \frac{1}{\rho} \left[\frac{\sigma_2 - \sigma_1}{\Delta x} + \frac{\sigma_4 - \sigma_3}{\Delta y} + \frac{\sigma_6 - \sigma_5}{\Delta z} \right] + g \quad (4)$$

but I don't see how this difference in stress vectors satisfies the force balance.

It seems like there's just a sign convention I'm missing, but I don't know what kind of sign convention satisfies both eq (4) and eq (3).