

Looking for equation for an ideal V source with a step V driving ideal capacitor. Both ideal components. The equation showing capacitor current.

For **ideal components** (ideal voltage source, ideal capacitor, zero resistance/inductance), the result is mathematically simple—and a bit extreme.

Setup

Let the voltage source apply a **step** at $t = 0$:

$$v(t) = V_0 u(t)$$

where $u(t)$ is the unit step function.

Capacitor current equation

For an ideal capacitor,

$$i(t) = C \frac{dv(t)}{dt}$$

Taking the derivative of the step:

$$\frac{d}{dt}[V_0 u(t)] = V_0 \delta(t)$$

So the capacitor current is:

$$i(t) = CV_0 \delta(t)$$

Interpretation

- The current is an **impulse** at $t = 0$
- Infinite peak current, zero duration

- Zero current for all $t \neq 0$
- Total charge delivered:

$$\int_{-\infty}^{\infty} i(t) dt = CV_0$$

which matches:

$$Q = CV_0$$

Laplace-domain form (often useful)

$$V(s) = \frac{V_0}{s}$$

$$I(s) = CsV(s) = CV_0$$

Constant in $s \rightarrow$ impulse in time.

Physical note

This impulse current only exists because **both elements are ideal**.

Any real circuit resistance or source impedance turns the impulse into a large but finite transient current.

If you want, I can also show:

- How this changes with a series R
- How to model it in SPICE
- Energy flow during the step



Different problem, a II components ideal. A step V driving a Capacitor that is in series with another capacitor and it paralld by a resistor. Looking for equations for V on each capacitor.

I'll assume the circuit is this (tell me if the topology is different):

- Ideal step voltage source: $v_s(t) = V_0 u(t)$
- Series capacitor C_1
- Node x
- From node x to ground: capacitor C_2 in parallel with resistor R

So:

source $\rightarrow C_1 \rightarrow$ node $x \rightarrow (C_2 \parallel R) \rightarrow$ ground

Variables

- $v_1(t)$: voltage across C_1
- $v_2(t)$: voltage across C_2 (this is also the node voltage)
- Capacitors initially uncharged: $v_1(0^-) = v_2(0^-) = 0$

By KVL:

$$v_1(t) + v_2(t) = V_0 u(t)$$

Currents

Series current through C_1 :

$$i(t) = C_1 \frac{dv_1}{dt}$$

Parallel currents at node x :

$$i(t) = C_2 \frac{dv_2}{dt} + \frac{v_2}{R}$$

Governing differential equation

Using $v_1 = V_0 - v_2$:

$$C_1 \frac{d}{dt}(V_0 - v_2) = C_2 \frac{dv_2}{dt} + \frac{v_2}{R}$$

For $t > 0$, V_0 is constant, so:

$$-(C_1) \frac{dv_2}{dt} = C_2 \frac{dv_2}{dt} + \frac{v_2}{R}$$

$$(C_1 + C_2) \frac{dv_2}{dt} + \frac{v_2}{R} = 0$$

Solution for $v_2(t)$

This is a first-order system:

$$\frac{dv_2}{dt} = -\frac{1}{R(C_1 + C_2)} v_2$$

$$\boxed{v_2(t) = v_2(0^+) e^{-t/\tau}} \quad \text{where } \tau = R(C_1 + C_2)$$

Initial condition $v_2(0^+)$

At $t = 0^+$, the resistor cannot conduct impulse current, so the step divides **capacitive-only**:

$$v_2(0^+) = V_0 \frac{C_1}{C_1 + C_2}$$

$$v_1(0^+) = V_0 \frac{C_2}{C_1 + C_2}$$

Final equations (main result)

Voltage on C_2 :

$$\boxed{v_2(t) = V_0 \frac{C_1}{C_1 + C_2} e^{-t/[R(C_1+C_2)]}}$$

Voltage on C_1 :

$$\boxed{v_1(t) = V_0 - v_2(t)}$$

Long-time behavior

$$v_2(\infty) = 0 \quad \Rightarrow \quad v_1(\infty) = V_0$$

All charge on C_2 leaks through R ; C_1 ends up holding the full source voltage.

If you want, I can also:

- Derive this using Laplace transforms
- Show the impulse current at $t = 0$
- Handle the case where the resistor is on the **other** capacitor



What is the voltage each capacitor at $t = 0$

At $t = 0^+$ (immediately after the step is applied):

- Both capacitors were uncharged at $t = 0^-$
 - The resistor **cannot conduct impulse current**
 - So the step voltage divides **only by capacitive charge sharing**
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Result (key answer)

For a step V_0 applied to C_1 in series with C_2 :

$$v_{C_1}(0^+) = V_0 \frac{C_2}{C_1 + C_2}$$

$$v_{C_2}(0^+) = V_0 \frac{C_1}{C_1 + C_2}$$

(The resistor in parallel with C_2 does **not** affect the instant $t = 0^+$.)

Why this happens (short explanation)

- Capacitor voltage cannot change instantaneously **unless driven by an impulse**
- The ideal step causes an impulse current through the series capacitors
- That impulse distributes charge such that:

$$Q = C_1 v_{C_1} = C_2 v_{C_2}$$

with:

$$v_{C_1} + v_{C_2} = V_0$$

Solving gives the results above.

Sanity checks

- Larger capacitor → **smaller voltage**
 - If $C_1 = C_2$: each gets $V_0/2$
 - If $C_2 \rightarrow \infty$: $v_{C_2}(0^+) \rightarrow 0$
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If you want, I can also show:

- The impulse current explicitly
- Energy distribution at $t = 0$
- What changes if one capacitor has an initial voltage



But at $t = 0^+$ the R robs some of the current delivered by C_1 so C_2 gets less current,

This is a very common (and reasonable) intuition—but **for an ideal step at exactly $t = 0^+$** , the resistor **does not** rob any of the charge that sets the capacitor voltages.

Here's the key point that resolves the confusion.

1. What happens at $t = 0^+$

The ideal step voltage produces an **impulse current** through the series capacitors. That impulse is what instantaneously establishes the capacitor voltages.

- Capacitors **can** carry impulse current
- A resistor **cannot**

Why? Because resistor current is:

$$i_R(t) = \frac{v(t)}{R}$$

To absorb any **finite charge** at an instant, the resistor would need an **impulse voltage**, which it does not have. The node voltage is finite.

2. Charge argument (most important)

The voltage at $t = 0^+$ is determined by **charge balance over the impulse**, not by ordinary current.

Charge into node x :

$$\int i_{C1}(t) dt = Q$$

Charge out of node x :

$$\int i_{C2}(t) dt + \int i_R(t) dt$$

But:

$$\int i_R(t) dt = \int \frac{v_2(t)}{R} dt = 0$$

because $v_2(t)$ is finite \rightarrow no impulse \rightarrow zero area.

So all impulse charge goes into C_2 .

3. Therefore, at $t = 0^+$

The resistor draws **zero charge**, and the situation is exactly the same as if it were not there.

$$Q = C_1 v_{C1}(0^+) = C_2 v_{C2}(0^+)$$

$$v_{C1}(0^+) + v_{C2}(0^+) = V_0$$

Solving:

$$v_{C1}(0^+) = V_0 \frac{C_2}{C_1 + C_2}$$

$$v_{C2}(0^+) = V_0 \frac{C_1}{C_1 + C_2}$$

4. When *does* the resistor matter?

Immediately **after** $t = 0^+$:

- The impulse is over
- Capacitor currents become finite
- The resistor begins drawing current
- $v_{C2}(t)$ decays exponentially

So your intuition is correct for $t > 0^+$, but not at the instant of the step.

5. Short intuition summary

- **Impulse event (0^+):** resistor invisible
- **Finite-time behavior:** resistor dominates decay
- **Voltage division at 0^+ :** purely capacitive

If you want, I can also:

- Draw the impulse-current waveform
- Show this rigorously in Laplace domain
- Show what changes if the source has series resistance



Why can an impulse deliver current to a C, who has time dependent hence V dependencies, but not to a R who has no time dependencies at the time current is being delivered to the capacitor

Great question. The clean answer comes down to **constitutive laws** and how they behave under **distributions (impulses)**.

I'll keep this rigorous but intuitive.

1. Constitutive laws

Capacitor

$$i_C(t) = C \frac{dv(t)}{dt}$$

Resistor

$$i_R(t) = \frac{v(t)}{R}$$

These look similar, but they behave **very differently** when $v(t)$ contains a step.

2. What an impulse actually is

An impulse $\delta(t)$ is the derivative of a step:

$$\frac{d}{dt} u(t) = \delta(t)$$

It has:

- infinite amplitude
 - zero duration
 - finite area
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3. Capacitor + step voltage

If:

$$v(t) = V_0 u(t)$$

then:

$$i_C(t) = C \frac{dv}{dt} = CV_0 \delta(t)$$

✓ The capacitor **naturally produces an impulse current** because its law involves a **time derivative**.

The impulse transfers charge:

$$Q = \int i_C dt = CV_0$$

4. Resistor + step voltage

For the same voltage:

$$i_R(t) = \frac{V_0}{R} u(t)$$

This is:

- finite
- non-impulsive
- no infinite spike

Charge delivered to the resistor:

$$Q_R = \int i_R dt$$

Over any infinitesimal interval around $t = 0$:

$$\lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{+\epsilon} \frac{V_0}{R} u(t) dt = 0$$

✗ No impulse → **no instantaneous charge transfer**

5. Key distinction (the core reason)

Capacitor law contains a derivative

- A finite step in voltage \Rightarrow impulse in current

Resistor law is algebraic

- A finite voltage \Rightarrow finite current
- Finite current over zero time \Rightarrow zero charge

This has nothing to do with “time dependence” in a casual sense; it’s about **differentiation vs multiplication**.

6. Why the resistor cannot “steal” impulse charge

At the node:

$$i_{C1} = i_{C2} + i_R$$

Integrate over the impulse interval:

$$\int i_{C1} dt = \int i_{C2} dt + \int i_R dt$$

But:

$$\int i_R dt = 0$$

So all impulse charge must go into the capacitors.

7. Physical intuition (non-mathy)

- A capacitor stores **charge**
 - An impulse is **pure charge transfer**
 - A resistor dissipates **power**, not charge
 - Power requires **time**
 - Zero time \Rightarrow zero energy dissipation \Rightarrow zero resistor effect
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8. One-line takeaway

Elements whose laws contain time derivatives can respond impulsively; algebraic elements cannot.

That's why impulses affect capacitors (and inductors), but not resistors.

If you want, I can also:

- Show this using Laplace transforms rigorously
- Discuss what breaks when components are non-ideal
- Connect this to distribution theory explicitly

