Compute $\int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x}\,dx \qquad 0 < a < 1$

$e^{ax} = t$

differentiate the equation

$ae^{ax} = dt$

$\therefore\ e^{ax} = \frac{1}{a}dt$

$e^x = t^{\frac{1}{a}}$

$$\int_{-\infty}^{\infty} \frac{\frac{1}{a}}{1+t^{\frac{1}{a}}}\,dt = \left[\frac{1}{a}\ln(1+\sqrt[a]{t})\right]_{-\infty}^{\infty}$$

$$\lim_{b\to-\infty}\left[\frac{1}{a}\ln(1+t^{\frac{1}{a}})\right]_b^1 + \lim_{b\to\infty}\left[\frac{1}{a}\ln(1+t^{\frac{1}{a}})\right]_1^b$$

$$\lim_{b\to-\infty}\left[\frac{1}{a}\ln 2 - \frac{1}{a}\ln(1+b^{\frac{1}{a}})\right] + \lim_{b\to\infty}\left[\frac{1}{a}\ln(1+b^{\frac{1}{a}}) - \frac{1}{a}\ln 2\right]$$

$$\lim_{b\to\infty}\left[\cancel{\frac{1}{a}\ln 2} - \frac{1}{a}\ln(1+(-b)^{\frac{1}{a}}\right] + \lim_{b\to\infty}\left[\frac{1}{a}\ln(1+b^{\frac{1}{a}}) - \cancel{\frac{1}{a}\ln 2}\right]$$

$$\lim_{b\to\infty} -\frac{1}{a}\ln(1+(-b)^{\frac{1}{a}} + \frac{1}{a}\ln(1+b^{\frac{1}{a}})$$

$$\lim_{b\to\infty} -\frac{1}{a}\left(\ln\frac{1+(-b)^{\frac{1}{a}}}{1+b^{\frac{1}{a}}}\right)$$

$$\lim_{b\to\infty} -\frac{1}{a}\left(\ln\frac{\frac{1}{b}+\frac{(-b)^{\frac{1}{a}}}{b}}{\frac{1}{b}+\frac{b^{\frac{1}{a}}}{b}}\right)$$

$$\lim_{b\to\infty} -\frac{1}{a}\left(\ln\frac{\frac{1}{b} - (-b)^{\frac{1}{a}-1}}{\frac{1}{b} + b^{\frac{1}{a}-1}}\right.$$

$$\lim_{b\to\infty} -\frac{1}{a}\left(\ln\frac{\cancel{\frac{1}{b}} - (-b)^{\frac{1}{a}-1}}{\cancel{\frac{1}{b}} + b^{\frac{1}{a}-1}}\right) \quad \lim_{b\to\infty}\frac{1}{a}\left(\frac{(-b)^{\frac{1}{a}-1}}{b^{\frac{1}{a}-1}}\right)$$

$\therefore\ \frac{1}{a}$