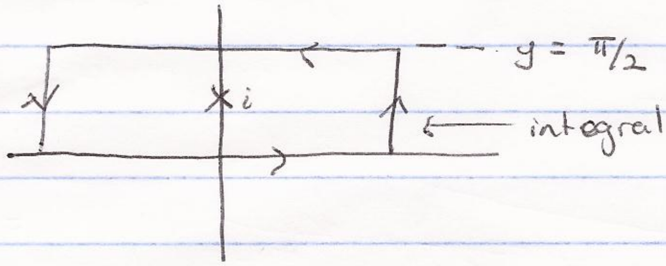
$$\frac{e^{az} - e^{-az}}{e^{\pi z} - e^{-\pi z}}$$

$$\int_{-\infty}^{\infty} \frac{\sinh ax}{\sinh \pi x}\, dx$$

$$\frac{1}{2}\left(e^{i\pi} + e^{-i\pi}\right) =$$
$$\frac{1}{2}(-1 + -1) = \frac{1}{2} \cdot -2$$

poles @ $z = in$, n is an integer

Draw box contour



integral goes → 0 along vert. edges as $R \to \infty$

$$\text{Res}[z = i] : \lim_{z \to i} \frac{(\sinh(az))(z - i)}{\sinh(\pi z)}$$

$$= \lim_{z \to i} \frac{\sinh(az) + (z-i)\, a\cosh(az)}{\pi \cosh(\pi z)} = \frac{\sinh(ia)}{\cosh(\pi i)} = -\sinh(ia)$$

$$\cancel{\int_{-\infty}^{\infty} \frac{\sinh(ax)}{\sinh(\pi x)}\, dx + \int_{\infty}^{-\infty} \frac{\sinh(ax)\, e^{i\pi/2}}{\sinh(\pi x)}\, dx} = -\sinh(ia)$$

?

$$\frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{ax} - e^{-ax}}{\sinh(\pi x)}\, dx + \frac{1}{2}\int_{\infty}^{-\infty} \frac{e^{i\pi/2 a} e^{ax} - e^{-i\pi/2 a} e^{-ax}}{\sinh(\pi x)}\, dx = -\sinh(ia)$$